1. Forward and Futures

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For the first three chapters and the midterm exam, I will focus on payoffs and pricing.

For payoffs, I want you to be familiar with plotting (and understanding) the payoffs of bonds, stocks (or other spot contracts such as currencies, commodities), forwards, European options, and their various combinations (portfolios).

Another related term I like to use is “exposures.” By being long or short in these contracts, you are essentially gaining (or hedging away) exposures to certain underlying movements. For example, if you are long a stock, you have a positive exposure to the stock price movement (that is, your portfolio’s value increases if the stock price goes up). Accordingly, I want you to think about when you want what “exposure” and how you achieve these exposures through forming portfolios of these different contracts. For example, if you think the stock price will go up, you will want to gain long exposure to the stock. You can do so by either long the stock (hold positive shares of the stock in your portfolio) or you can be long in a forward contract on this stock. Then, I want you to become exact on how much your portfolio will be under what situations — and that is essentially the payoff plot.

For pricing, the focus is to price a contract or a portfolio of contracts based on (i) their payoff structures and (ii) the prices (costs) of contracts with which you can generate such payoff structures. It is never about forecasted cash flow (e.g. analysts forecasts or inside information), and it is always about pricing the contract based on the cost of constructing the payoff structure using a portfolio of other contracts that you know value of.

Given this focus, for chapter 1, I will not spend time on the institutional differences between forwards and futures. They will be treated as the same in all homeworks and exams. I will skip the hedging with futures section. The focus is to understand the basic payoff structures of a spot contract, a bond, and a forward, and how to replicate (and price) a forward contract via “buy-and-carry” arguments.

For this homework, I denote today (now) as time 0, and the “judgement” date as $T$. That is, if we are dealing with contracts with a maturity date (expiration date) such as forwards and bonds, we choose these contracts to mature at time $T$. If we are dealing with contracts that do not have a maturity date (such as currencies and stocks), we are only concerned about their prices at this particular date.

We consider only one “underlying” risky security (it can be a stock or exchange rate), and we use $S$ to denote its price, with $S_0$ being its current price (known) and $S_T$ being its price at time $T$ (unknown). Also, ignore bid-ask spread and transaction cost and assume that you can buy or sell any amount of the security at the price $S_0$. To be concrete, let’s set $S_0 = 100$.

1. Suppose you are long one share (you own one share) of this underlying security, plot your portfolio’s payoff at time $T$. Assume that the security does not pay dividend or earn interest.

2. Suppose you are long one share (you own one share) of this underlying security, plot your portfolio’s payoff at time $T$. Assume that the security pays $5 dividend at $T$ (and nothing in between).

3. Suppose you are long ten shares of this underlying security (and nothing else), plot your portfolio’s payoff at time $T$. How much shall you sell this portfolio for? Assume that the security earns continuously compounded rate of 5% per year and $T = 2$ years.
4. Plot the time-$T$ payoff of a zero-coupon bond with maturity date $T$ and par value of $1$ as a function of this underlying security price at time $T$. Also, if the continuously compounding interest rate is 5% per year, how much is this bond worth today is $T = 2$?

5. Suppose you borrow $S_0 = 100$ dollars to buy one share of this security today, the lending rate on the dollar is 5% per year (continuous compounding). The security does not pay dividend or earning interest. Plot the time-$T$ payoff of your portfolio. Also explain what your portfolio is composed of.

   (a) If there are forward contracts traded on this security with the expiry $T = 2$, what should be the fair forward price?

   (b) Design a trading strategy to make money if the forward price is $105$ or $115$, respectively.

   (c) Suppose your portfolio is just long a forward contract expiring at $T = 2$ and with a delivery price $(K)$ of $100$, what is the value of your portfolio? What’s the value if the delivery price is $120$?

6. Repeat the question, but assume that the security also earns a continuously compounding interest rate of 2%.

7. Repeat the question, but assume that the security has quarterly dividend payments of $2$ for each share. And assume that you are at the beginning of a quarter so that you’ll have 8 dividend payments over the next two years.