Options Trading Strategies

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Options Markets
Objectives

A strategy is a set of options positions to achieve a particular risk/return profile.

For simplicity, we focus on strategies that involve positions in only European options on the same underlying and at the same expiration.

The zero-coupon bond and the underlying forward of the same maturity are always assumed available.

We hope to achieve three objectives:

1. Given a strategy (a list of derivative positions), we can figure out its risk profile, i.e., the payoff of the strategy at expiry under different market conditions (different underlying security price levels).

2. Given a targeted risk profile at a certain maturity (i.e., a certain payoff structure), we can design a strategy using bonds, forwards, and options to achieve this profile.

3. Be familiar with (the risk profile, the objective, and the composition of) the most commonly used, simple option strategies, e.g., straddles, strangles, butterfly spreads, risk reversals, bull/bear spreads.
Put-call conversions

Plot the payoff function of the following combinations of calls/puts and forwards at the same strike $K$ and maturity $T$.

1. Long a call, short a forward.
   - Compare the payoff to long a put.

2. Short a call, long a forward.
   - Compare the payoff to short a put.

3. Long a put, long a forward.
   - Compare the payoff to long a call.

4. Short a put, short a forward.
   - Compare the payoff to short a call.

5. Long a call, short a put.
   - Compare the payoff to long a forward.

6. Short a call, long a put.
   - Compare the payoff to short a forward.
Put-call conversions: Payoff comparison ($K = 100$)

The dash and dotted lines are payoffs for the two composition instruments. The solid lines are payoffs of the target.
The linkage between put, call, and forward

- The above conversions reveal the following parity condition in payoffs of put, call, and forward at the same strike and maturity:

  Payoff from a call \(-\) Payoff from a forward \(=\) Payoff from a put
  Payoff from a put \(+\) Payoff from a forward \(=\) Payoff from a call
  Payoff from a call \(-\) Payoff from a put \(=\) Payoff from a forward

- If the payoff is the same, the present value should be the same, too (put-call parity):

\[ c_t - p_t = e^{-r(T-t)}(F_{t,T} - K). \]

- At a fixed strike \((K)\) and maturity \(T\), we only need to know the two prices of the following three: \((c_t, p_t, F_{t,T})\).

  One of the three contracts is redundant.
In the absence of forward, use spot and bond:

- Can you use a spot and bond to replicate a forward payoff?
- What’s the payoff function of a zero bond?
**Popular payoff I: Bull spread**

Can you generate the above payoff structure (solid blue line) using (in addition to cash/bond):

- two calls
- two puts
- a call, a put, and a stock/forward

Who wants this type of payoff structure?
Generating a bull spread

- **Two calls:** Long call at $K_1 = 90$, short call at $K_2 = 110$, short a bond with $10$ par.

- **Two puts:** Long a put at $K_1 = 90$, short put at $K_2 = 110$, long a bond with $10$ par.

- **A call, a put, and a stock/forward:** Long a put at $K_1 = 90$, short a call at $K_2 = 110$, long a forward at $K = 100$ (or long a stock, short a bond at $100$ par).
Pointers in replicating payoffs

- Each kinky point corresponds to a strike price of an option contract.
- Given put-call party, you can use either a call or a put at each strike point.
- Use bonds for parallel shifts.
- A general procedure using calls, forwards, and bonds
  - Starting from the left side of the payoff graph at \( S_T = 0 \) and progress to each kinky point sequentially to the right.
  - If the payoff at \( S_T = 0 \) is \( x \) dollars, long a zero-coupon bond with an \( x \)-dollar par value. [Short if \( x \) is negative].
  - If the slope of the payoff at \( S_T = 0 \) is \( s_0 \), long \( s_0 \) shares of a call/forward with a zero strike — A call at zero strike is the same as a forward at zero strike. [Short if \( s_0 \) is negative.]
  - Go to the next kinky point \( K_1 \). If the next slope (to the right of \( K_1 \) is \( s_1 \), long \( (s_1 - S_0) \) shares of call at strike \( K_1 \). Short when the slope change is negative.
  - Go to the next kinky point \( K_2 \) with a new slope \( s_2 \), and long \( (s_2 - s_1) \) shares of calls at strike \( K_2 \). Short when the slope change is negative.
  - Keep going until there are no more slope changes.
Pointers in replicating payoffs, continued

- A general procedure using puts, forwards, and bonds
  - Starting from the right side of the payoff graph at the highest strike under which there is a slope change. Let this strike be $K_1$.
  - If the payoff at $K_1$ is $x$ dollars, long a zero-coupon bond with an $x$-dollar par value. [Short if $x$ is negative].
  - If the slope to the right of $K_1$ is positive at $s_0$, long $s_0$ of a forward at $K_1$. Short the forward if $s_0$ is negative.
  - If the slope to the left of $K_1$ is $s_1$, short $(s_1 - s_0)$ shares of a put at $K_1$. Long if $(s_1 - s_0)$ is negative.
  - Go to the next kinky point $K_2$. If the slope to the left of $K_2$ is $s_2$, short $(s_2 - s_1)$ put with strike $K_2$.
  - Keep going until there are no more slope changes.
Example: Bear spread

- How many (at minimum) options do you need to replicate the bear spread?
- Do the exercise, get familiar with the replication.
- Who wants a bear spread?
Example: Straddle

- How many (at minimum) options do you need to replicate the straddle?
- Do the exercise, get familiar with the replication.
- Who wants long/short a straddle?
How many (at minimum) options do you need to replicate the strangle?

Do the exercise, get familiar with the replication.

Who wants long/short a strangle?
Example: Risk Reversal

- How many (at minimum) options do you need to replicate the risk reversal?
- Do the exercise, get familiar with the replication.
- Who wants long/short a risk reversal?
How many (at minimum) options do you need to replicate the butterfly spread?

Do the exercise, get familiar with the replication.

Who wants long/short a butterfly spread?
Suppose you construct a butterfly with the center strike at $100, and with the two side strikes at $99 and $101. Then, you will get paid $1 when the stock price reaches $100 at expiry, and paid nothing if the stock price is either below $99 or above $101.

The price of the butterfly reflects the “risk-adjusted” probability that the stock price will fall between (99,101), times the present value of one dollar (discount).

You can construct such butterflies with center strikes at $80,$81, $119, $120, ...

The cost/price of each fly reflects the probability of the stock price falling around the center strike of that fly.

Thus, if you have options at all strikes, you can construct these butterflies and infer the probabilities of the future stock price reaching each price level.

Breeden and Litzenberger (1978) for the underlying theory, and and many following papers on practical implementation...
Smooth out the kinks: Can you replicate this?

How many options do you need to replicate this quadratic payoff?

- You need a continuum of options to replicate this payoff.
- The weight on each strike $K$ is $2dK$.

Who wants long/short this payoff?

- The variance of the stock price is $\mathbb{E}[(S_T - F_{t,T})^2]$.
- Variance swap contracts on major stock indexes are actively traded.
Replicate any terminal payoff with options and forwards

\[
f(S_T) = f(F_t) + f'(F_t)(S_T - F_t) + \left\{ \begin{array}{l} \int_{0}^{F_t} f''(K)(K - S_T)^+ dK \\ \int_{F_t}^{\infty} f''(K)(S_T - K)^+ dK \end{array} \right\} \]

- What does this formula tell you?
  - With bonds, forwards, and European options, we can replicate any terminal payoff structures.
  - More exotic options deal with path dependence, correlations, etc.

- You do not need to memorize the formula.

VIX—CBOE’s Volatility Index

- It is meant to capture the expected annualized volatility of the S&P 500 Index return over the next 30 days.
- It is created as the weighted average price of 30-day S&P 500 Index options across all strikes, with the weighting proportional to $1/K^2$.