Market Anticipation of Fed Policy Changes and the Term Structure of Interest Rates

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Abstract. The Federal Reserve adjusts the federal funds target rate discretely, causing discontinuity in short-term interest rates. Unlike Poisson jumps, these adjustments are well anticipated by the market. We propose a term structure model that incorporates an anticipated jump component with known arrival times but random jump size. We find that doing so improves the model performance in capturing the term structure behavior. The mean jump sizes extracted from the term structure match the realized target rate changes well. Specification analysis indicates that the jump sizes show strong serial dependence and dependence on the interest-rate factors.

JEL Classification: E43, G12, G13, C51

1. Introduction

The Federal Reserve implements its monetary policies mainly through adjustments to the federal funds rate target. These adjustments are discrete in time, inducing discontinuous movements in short-term interest rates. Many studies model the federal funds rate target movements by using random compound Poisson jumps, where the Poisson arrival rate is potentially time varying and state dependent. These models capture the unexpected element of the federal funds rate target adjustment, i.e., Fed policy surprises. However, as Johannes (2004) points out, although incorporating a random jump component helps improve the statistical fit of the interest-rate time-series dynamics, this random jump has little impact on the term structure of interest rates.

Historical experience indicates that the federal funds rate target movement deviates from a random Poisson jump in at least two important ways. First, many of the federal funds rate target adjustments are not surprises, but are well anticipated by the market. There is a growing consensus among monetary economists that
monetary policy becomes effective not through its control of the current federal funds rate level per se, but through its impact on the market’s expectation of how the short-term interest rate is going to evolve in the future (Woodford (2003)). In line with this consensus, the decision making process in the Federal Reserve has become increasingly transparent in hope of a higher impact on market expectation (Bernanke (2004a)). The market, in turn, has been receiving increasingly less surprises from the Federal Reserve. Second, the anticipated move on the federal funds rate has an obvious impact on the term structure of interest rates. As the anticipated meeting date nears and as the anticipated size of the rate change increases, the whole term structure often becomes “distorted.” Significant differences arise between the term structure observed from the market and that implied from any dynamic term-structure models that assume pure diffusion or random-jump/diffusion dynamics. Earlier works on the federal funds target rate (Balduzzi, Bertola, and Foresi (1997) and Balduzzi, Bertola, Foresi, and Klapper (1998)) have also shown that the spreads between short-term rates and the overnight federal funds rate are mainly driven by expectations of future changes in the federal funds target rate.

To capture the frequent federal funds rate target adjustment and to better explain a term structure that is highly influenced by these well-anticipated federal reserve moves, we propose a dynamic term structure model with an anticipated jump component. This jump component arrives at known times but with random jump size. We fix the variance of the jump size, but allow the conditional mean jump size to vary over time to capture the market’s continuously updating belief about future target changes. We estimate the model using the U.S. dollar LIBOR and swap rates and compare its performance to that of a benchmark model without the anticipated jump component. We find that incorporating this anticipated jump component significantly improves the ability of the model to capture the term structure of interest rates at both short and long maturities.

Our estimated model generates a time series of the conditional mean jump sizes from the observed term structures. Comparing this implied mean jump size to the actual federal funds rate target changes, we find that historically, the yield curve and hence the interest rate market anticipate the federal funds rate movement very well. Further specification analysis indicates that the anticipated jump sizes show strong serial dependence, as found in Balduzzi, Bertola, and Foresi (1997). We also find that the anticipated jump sizes depend negatively on the three interest rate factors, all of which load positively on the interest rate term structure. This factor dependence suggests that the Federal Reserve is more likely to cut rates when interest rates are high.

The paper is structured as follows. Section 2 surveys the related literature and provides some background information on the FOMC operating procedure, and in particular its communications with the market. Section 3 describes the design of a three-factor dynamic term structure model with a jump component that captures
the anticipated move by the Federal Reserve. Section 4 describes the data set and the estimation methodology. Section 5 analyzes the model’s parameter estimates and performance, and the impacts of the jump component on the mean yield curve and factor loading. We also compare the anticipated jump size with the actual federal funds target rate changes during the past decade. Section 6 performs further specification analysis on how the jump sizes depend on past Fed policy changes and the current state of the economy. Section 7 concludes.

2. Background

During the past decade, the Federal Reserve has become much more transparent in its views and its decision-making processes. From January 1989 to December 1993, the FOMC relied on open market operations, rather than statements, to signal shifts in the stance of monetary policy. From February 1994 to November 1998, the FOMC began releasing statements that accompanied changes in the federal funds rate. Those statements offered a brief description of the rationale for the policy action. In December 1998, the FOMC implemented another important change in its disclosure practices. In addition to releasing statements to accompany policy actions, the FOMC decided to release statements when it wanted to communicate to the public a major shift in its views about the balance of risks or the likely direction of future policy. FOMC disclosure policy changed again in January 2000, when the Committee announced that a statement would be released after every FOMC meeting. This statement would always include an assessment of the “balance of risks.” This balance-of-risks assessment involves new language linked more closely to the Committee’s macroeconomic objectives than to the near-term direction of policy. As a result of this increasing transparency, the market is better able to anticipate the federal funds rate target changes. Kohn and Sack (2003) provide a detailed review on the evolution of the communication of the FOMC to the public. More recently, Federal Reserve Board Governor Ben S. Bernanke (2004) elaborates on why central bank talk, or Fedspeak, can make monetary policy more effective.

We analyze the impact of anticipated federal funds rate target moves within the affine term structure framework. Classic one-factor examples of affine term structure models include Vasicek (1977) and Cox, Ingersoll, and Ross (1985). Duffie and Kan (1996) characterize the complete class when the underlying state vector follows diffusion dynamics. Duffie, Pan, and Singleton (2000) consider the extended case in which the state dynamics contain random compound Poisson jumps with potentially state-dependent Poisson arrival rates. Piazzesi (2001) derives the bond pricing relation in the presence of a deterministic jump component under an affine-quadratic framework. Our deterministic jump specification follows the theoretical work of Piazzesi. However, our model design represents a strong
simplification and abstraction of her general framework that is necessary for the model to be practically implementable. To the best of our knowledge, our paper is the first to estimate a dynamic term structure model with deterministic jumps.

Within the affine class, we choose as our benchmark a three-factor Gaussian affine model with affine market price of risk. Using dimension of three is consistent with the empirical evidence of Litterman and Scheinkman (1991), Knez, Litterman, and Scheinkman (1994), and Heidari and Wu (2003). It is also the choice of recent empirical works on model designs, e.g., Backus, Foresi, Mozumdar, and Wu (2001), Dai and Singleton (2000, 2002, 2003), and Duffee (2002). Our choice of Gaussian variables with affine market price of risk follows Duffee (2002) and Dai and Singleton (2002). Both studies find that the combination of Gaussian variables and affine market price of risk specification is essential to explaining the behavior of bond excess returns and the expectation hypothesis regression coefficients.

Johannes (2004) identifies the strong presence of jumps in the interest-rate time-series dynamics, but he focuses on the surprise element of the interest-rate jump and only considers random Poisson jumps in the model specification. He finds that the random Poisson jumps have little impact on the term structure of the yield curve. In contrast, we focus on the anticipated component of the Federal Reserve policy change.

Piazzesi (2005) explicitly models the federal funds target rate as a pure random jump process with strongly time and state-dependent arrival rates. She assumes large, state-dependent jump arrival rates around the FOMC meeting dates, but a small, constant jump arrival rate in between the FOMC meeting dates. Piazzesi captures the market anticipation of the target rate changes via a strongly time and state-dependent arrival rate specification. In contrast, we adopt a deterministic jump specification. Our modeling structure provides a parsimonious and effective alternative in capturing market anticipation of target rate changes and its impact on the term structure.

Also related to our work is the recent literature on monetary policies. By combining microeconomic foundations with sticky price assumptions, monetary economists have derived a new generation of models on optimal monetary policy decisions. Clarida, Gali, and Gertler (1999) provide an excellent survey of the literature. Woodford (2003) provides a comprehensive analysis of the topic in his recent book, Interest and Prices. The key consensus from this literature is that the effectiveness of monetary policy lies in its impact on market expectations. The growing transparency in the decision-making processes of many major central banks reflects the intentions of these banks to obtain greater impact on market expectation (Bernanke (2004a) and Bernanke and Reinhart (2004)).

In other related works, Babbs and Webber (1996) and Farnsworth and Bass (2003) derive theoretical models that link aspects of monetary policy to the term structure. Das (2001), Hamilton (1996), and Hamilton and Jordá (2002) analyze the
time-series dynamics of the federal funds rate. Balduzzi, Bertola, and Foresi (1997), Balduzzi, Bertola, Foresi, and Klapper (1998), and Rudebusch (1995) estimate the short-rate dynamics and compute long yields using expectations hypothesis. Romer and Romer (2000) test the role played by asymmetric information in the relation between the Federal Reserve policy changes and changes on long-term interest rates. Kuttner (2001) decomposes the federal funds rate changes into two components, the surprise element and the anticipated component. He analyzes their different impacts on bond returns. Cochrane and Piazzesi (2002), Cook and Hahn (1989), and Evans and Marshall (1998) use regression analysis to study the relation between Federal Reserve policy changes and interest rates. Sarno and Thornton (2003) use cointegration analysis to investigate the dynamic link between the federal funds rate and the Treasury bill rate. They find that, contrary to the conventional monetary policy view that the federal funds rate “anchors” the short end of the U.S. money market, the federal funds rate actually lags behind the market. Adjustment toward the long-run equilibrium occurs through movements in the federal funds rate, rather than the other way around. Such findings are inconsistent with federal reserve “surprises,” but are consistent with our argument that most of federal funds rate adjustments are well anticipated by the market, and that the market anticipation shows up on the yield curve.

3. A Dynamic Term Structure Model with an Anticipated Jump

Within the affine framework, we first specify a three-factor benchmark model with pure diffusion dynamics. We then incorporate a deterministic jump component to the benchmark model and analyze its impact on the term structure.

3.1 A BENCHMARK GAUSSIAN-AFFINE MODEL

We fix a filtered complete probability space \( \{\Omega, \mathcal{F}, \mathbb{P}, (\mathcal{F}_t)_{0 \leq t \leq T}\} \) that satisfies the usual technical conditions in which \( T \) is some finite, fixed time. We use \( X \in \mathbb{R}^3 \) to denote a three-dimensional vector Markov process that represents the systematic state of the economy, and assume that the instantaneous interest rate \( r \) is affine in the state vector,

\[
r(X_t) = a_r + b_r^\top X_t, \tag{1}
\]

where the parameter \( a_r \in \mathbb{R} \) is a scalar and \( b_r \in \mathbb{R}^{3+} \) is a vector.

For the benchmark model, we assume that under the statistical measure \( \mathbb{P} \), the state vector is governed by a pure-diffusion Ornstein-Uhlenbeck (OU) process,

\[
dX_t = -\kappa X_t dt + dW_t, \tag{2}
\]
where $\kappa \in \mathbb{R}^{3 \times 3}$ controls the mean reversion of the vector process. With no loss of generality, we normalize the process to have zero long-run mean and identity instantaneous covariance matrix. Given the Gaussian nature of the state vector $X_t$, we constrain the short-rate loading coefficients $b_r$ to be positive for identification.

We close the model by assuming an affine market price of risk \[ \gamma(X_t) = b_\gamma + \kappa_\gamma X_t \] (3) with $b_\gamma \in \mathbb{R}^3$ and $\kappa_\gamma \in \mathbb{R}^{3 \times 3}$. The affine market price of risk specification dictates that the state vector $X_t$ remains Ornstein-Uhlenbeck under the risk-neutral measure $\mathbb{P}^*$, but with an adjustment to the drift term, \[ dX_t = (-b_\gamma - \kappa^* X_t)dt + dW^*_t, \quad \kappa^* = \kappa + \kappa_\gamma. \] (4)

The time-$t$ value of the zero-coupon bond with time-to-maturity $\tau$ can be written as a conditional expectation of future short rates under the risk-neutral measure $\mathbb{P}^*$, \[ P(X_t, \tau) = \mathbb{E}^*_{t}[\exp\left(-\int_t^{t+\tau} r(X_s) ds\right)], \] (5) where $\mathbb{E}^*_t[\cdot]$ denotes the expectation operator under measure $\mathbb{P}^*$ conditional on time-$t$ filtration $\mathcal{F}_t$. The expectation can be solved analytically as an exponential affine function of the time-$t$ state vector $X_t$, \[ P(X_t, \tau) = \exp(-a(\tau) - b(\tau)^T X_t), \] (6) where the coefficients $a(\tau)$ and $b(\tau)$ are determined by the following ordinary differential equations:

\[ a'(\tau) = a_r - b(\tau)^T b_\gamma - b(\tau)^T b(\tau)/2, \]
\[ b'(\tau) = b_r - (\kappa^*)^T b(\tau), \] (7)

subject to the boundary conditions $b(0) = 0$ and $c(0) = 0$. The ordinary differential equations can be solved analytically in terms of the eigenvalues and eigenvectors of $\kappa^*$.

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1 Let $y$ denote a general OU process,
\[ dy_t = (a + by_t)dt + \Sigma dW_t, \]
with general coefficients $a, b, \Sigma$, the process $X$ in (2) is obtained by the following affine transformation,
\[ X = \Sigma^{-1}(y - b^{-1}a), \]
where $\kappa = -\Sigma^{-1}b\Sigma$. Since the short rate $r$ is affine in $X$ as in (1), it is also affine in $y$ with a mere re-definition of the affine coefficients. Thus, our normalization in (2) does not lose any generality.
### 3.2 THE ANTICIPATED FEDERAL FUNDS RATE MOVE

To capture the anticipated federal funds rate move, we incorporate a deterministic jump component, $J_t$, to the third factor $X_3_t$. This last factor mainly captures the variation of short-term interest rates. Since we observe discontinuous movements mostly on short-term interest rates, it is appropriate to place the jump component in this last factor.

We use one anticipated jump component to summarize the impact of the anticipated federal funds rate target moves on the term structure. This jump component arrives at a fixed (deterministic) time horizon but with a random jump size. We use $\tau_J$ to denote the time between now (time $t$) and the arrival of the jump, and $a_J(t + \tau_J)$ to denote the random jump size at time $t + \tau_J$.

Given the one-jump assumption, the risk-neutral dynamics for the state vector $X$ remain the same as in equation (4) between time $t$ (now) and $t + \tau_J$. Therefore, the pricing solution remains the same as in equation (6) for zero-coupon bonds with time to maturities shorter than $\tau_J$.

For zero-coupon bonds that expire after $t + \tau_J$, we can regard the state dynamics as having two components, a random shift of $a_J(t + \tau_J)$ at a fixed time $t + \tau_J$ and a diffusion dynamics as described in equation (4). We can rewrite the bond pricing equation in (5) as,

$$P(X_t, \tau) = \mathbb{E}_t^*[\exp\left(-\int_t^{t+\tau_J} r(X_s)\, ds\right)] \mathbb{E}_{t+\tau_J}^*[\exp\left(-\int_{t+\tau_J}^T r(X_s)\, ds\right)].$$

(8)

Since there is no jump before $t + \tau_J$, we have

$$\mathbb{E}_t^*[\exp\left(-\int_t^{t+\tau_J} r(X_s)\, ds\right)] = e^{-a(t+\tau_J) - b(t+\tau_J)^\top X_t},$$

(9)

where $[a(t+\tau_J), b(t+\tau_J)]$ follow the same ordinary differential equation as in (7).

Furthermore, conditioning on the deterministic jump at time $t + \tau_J$, there are no more deterministic jumps in the future, hence the state dynamics follow equation (4) again. Thus, we have

$$\mathbb{E}_{t+\tau_J}^*[\exp\left(-\int_{t+\tau_J}^{T} r(X_s)\, ds\right)] = e^{-a(t+\tau_J) - b(t+\tau_J)^\top X_{t+\tau_J}},$$

(10)

where $X_{t+\tau_J}$ denotes the value of $X$ at time $t + \tau_J$ right after the jump. If we let $\bar{X}_{t+\tau_J}$ denote the value of $X$ at time $t + \tau_J$ just prior to the jump, we have

$$X_{t+\tau_J}(1:2) = \bar{X}_{t+\tau_J}(1:2), \quad X_{t+\tau_J}(3) = \bar{X}_{t+\tau_J}(3) + a_J(t + \tau_J),$$

(11)
as there is a jump component only in the third element of the state vector. Plugging (9), (10), and (11) into equation (8), we have

\[
\begin{align*}
P(X_t, \tau) &= \exp \left( -a(\tau - \tau_J) - b(\tau - \tau_J) \top \tilde{X}_{t+\tau_J} - b(\tau - \tau_J) \tau_J (t + \tau_J) \right) \\
&\quad \times \mathbb{E}_{\tau}^* \left[ e^{-a(\tau - \tau_J) - b(\tau - \tau_J) \top \tilde{X}_{t+\tau_J} - b(\tau - \tau_J) \tau_J (t + \tau_J)} \right],
\end{align*}
\]

(12)

where \(b(\tau - \tau_J)_3\) denotes the third element of the vector. Equation (12) shows that the bond pricing depends on the joint Laplace transform of the pre-jump state vector \(\tilde{X}_{t+\tau_J}\) and the random jump size \(\bar{a}_J(t + \tau_J)\) under the risk-neutral measure \(\mathbb{P}^*\). As long as the joint Laplace transform is exponential affine in the state vector, the zero-coupon bond prices remain as exponential affine functions of the state vector.

To further separate the effect of the jump component, we make the simplifying assumption that the random jump size \(\bar{a}_J(t + \tau_J)\) is independent of the pre-jump state vector \(\tilde{X}_{t+\tau_J}\). Then, we have

\[
P(X_t, \tau) = \left[ \exp \left( -a(\tau) - b(\tau) \top X_t \right) \right] \left[ \mathbb{E}_{\tau}^* \left( e^{-b(\tau - \tau_J) \bar{a}_J(t + \tau_J)} \right) \right].
\]

(13)

The term in the first bracket forms the bond pricing formula under the Gaussian-affine diffusion dynamics, as we have derived in the previous section. The impact of the deterministically arriving random jump component is captured by the terms in the second bracket, which is the Laplace transform of the random jump size \(\bar{a}_J(t + \tau_J)\) evaluated at the coefficient \(b(\tau - \tau_J)_3\) under the risk-neutral measure \(\mathbb{P}^*\). Thus, under the independence assumption, an exponential-affine bond pricing relation remains as long as the Laplace transform of the jump size under measure \(\mathbb{P}^*\) is exponential affine in \(X_t\).

To derive the Laplace transform of the random jump size \(\bar{a}_J(t + \tau_J)\), we further assume that right before the jump occurs at time \(t + \tau_J\), the jump size has a normal distribution under the risk-neutral measure \(\mathbb{P}^*\) with a variance of \(\sigma_J^2\), which we assume as a constant, and a mean of \(\bar{a}_J(t + \tau_J)\), which we assume that the market updates following a random walk under the statistical measure \(\mathbb{P}\),

\[
d\bar{a}_J^* (t) = \sqrt{q_J} dW_J^t,
\]

(14)

where the Brownian motion \(W_J^t\) is independent of the Brownian motions \(W_t^i\) in the state vector in equation (2). The risk-neutral counterpart of the updating dynamics in equation (14) can be written as,

\[
d\bar{a}_J^* (t) = -\eta_J dt + \sqrt{q_J} dW_J^t,
\]

(15)

where \(\eta_J\) represents a risk premium term induced by possible market pricing of \(W_J^t\) risk. Hence, conditional on time-\(t\) filtration \(\mathcal{F}_t\), the time-\(t\) mean jump forecast \(\bar{a}_J^* (t)\), and under the risk-neutral measure \(\mathbb{P}^*\), the jump size has a normal distribution with mean \(\bar{a}_J(t) = \bar{a}_J^* (t) - \eta_J \tau_J\) and variance \(\sigma_J^2 = \sigma_J^2 + \tau_J q\). Since our stylized model incorporates one jump with a fixed time horizon \(\tau_J\), we cannot separately identify \(\bar{a}_J^* (t)\) and \(\eta_J\), but can only identify the risk-neutral mean jump size \(\bar{a}_J(t)\)
over the fixed horizon $\tau_J$. We also directly estimate the jump variance $\sigma^2_J$ over the fixed horizon $\tau_J$.

Under the above assumptions on the jump size, we can carry out the expectation in (13) explicitly,

$$P(X_t, \tau) = \exp \left( -a(\tau) - b(\tau)^\top X_t - b(\tau - \tau_J) a_J(t) + \frac{1}{2} b(\tau - \tau_J)^2 \sigma^2_J \right).$$

(16)

Thus, if we use $[a^J(\tau, t), b^J(\tau)]$ to denote the affine coefficients for the model with jumps, we can link them to the coefficients without jumps $[a(\tau), b(\tau)]$ as,

$$a^J(\tau, t) = a(\tau) + b(\tau - \tau_J) a_J(t) - \frac{1}{2} b(\tau - \tau_J)^2 \sigma^2_J, \quad b^J(\tau) = b(\tau),$$

(17)

where the coefficients $b(\tau)$ are set to zero when $\tau \leq 0$ so that the relation holds across all maturities. Note that in the presence of the deterministically arriving jump, the bond pricing coefficients $a^J(\tau, t)$ at time $t$ are no longer time-homogeneous as they depend on the time-$t$ market anticipated value of the jump size $a_J(t)$.

In model estimation, we treat $a_J(t)$ as an additional dynamic factor that the market updates continuously. Under this treatment, we define the expanded state vector as,

$$Z_t \equiv [X_t^\top, a_J(t)]^\top.$$ The bond pricing equation can be rewritten as an exponential affine function of the expanded state vector,

$$P(X_t, \tau) = \exp(-c(\tau) - d(\tau)^\top Z_t),$$

(18)

with

$$c(\tau) = a(\tau) - \frac{1}{2} b(\tau - \tau_J)^2 \sigma^2_J, \quad d(\tau_{1:3}) = b(\tau), \quad d(\tau_4) = b(\tau - \tau_J).$$

(19)

Comparing the term structure of the benchmark model without jumps to this model with one anticipated jump of random size at $\tau_J$, we observe that the jump does not have an impact on the term structure for maturities shorter than the horizon of the anticipated jump arrival time, i.e. $\tau \leq \tau_J$. For rates maturing after the arrival of the anticipated jump, the yield curve after maturity $\tau_J$ is shifted by a deterministic amount, determined by $b(\tau - \tau_J) a_J(t) - \frac{1}{2} b(\tau - \tau_J)^2 \sigma^2_J$. The magnitude of the shift on the time-$t$ yield curve depends on the time-$t$ risk-neutral expected value of the jump size $a_J(t)$ and a convexity adjustment term due to the uncertainty in the jump size.

In reality, the Federal Reserve rarely adjusts its target rate by more than 50 basis points at a time. However, this practice does not mean that the market never expects

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2 Since 1995, there has never been target rate changes larger than 50 basis points. Such changes did occur before 1995. For example, on November 15, 1994, the federal reserve raised the federal funds target rate by 75 basis points.
a larger move. When the market expects the Federal Reserve to make a larger policy change, the Reserve often implements the larger policy change gradually via consecutive small steps (Bernanke (2004b)). We use one jump to succinctly summarize the aggregate impact of these consecutive small moves. Because of our one-jump aggregation, we need a horizon for this deterministic jump to be long enough to accommodate all the possible small-step moves. We set this horizon at three months. The FOMC has eight scheduled meetings each year, roughly every six weeks. A three-month horizon spans two to three FOMC regular meetings and serves as an appropriate horizon to summarize the consecutive small-step moves.

Our observation from the fed fund futures market also supports the choice of three months as an appropriate horizon. The market anticipation of the federal funds rate changes can often be inferred from the fed fund futures market. When the market expects a move in the near future, this move shows up as a difference between the fed fund futures rate and the overnight federal funds rate (Balduzzi, Bertola, and Foresi (1997) and Balduzzi, Bertola, Foresi, and Klapper (1998)). When the market expects consecutive target rate moves in the same direction, the expectation shows up in the relative differences between the fed fund futures at different maturities. The fed fund futures have maturities from one to six months, but the trading activity declines dramatically for maturities over three months. The variations in the differences between the fed fund futures rates of adjacent maturities also decline rapidly when the futures maturity is over three months. Hence, when the market participants anticipate large target rate changes, they expect it to happen within the next three months.

4. Data and Estimation

We estimate and compare the three-factor Gaussian affine model with and without an anticipated jump component, and gauge the economic significance of an anticipated federal funds rate move based on a panel data of U.S. dollar LIBOR and swap rates. We download the data from Bloomberg. The LIBOR have maturities of one, two, three, six, and 12 months, and swap rates have maturities of two, three, five, seven, ten, 15, and 30 years. The data are weekly (Wednesday) closing mid-quotes from January 4, 1995 to July 11, 2007 (654 observations).

The LIBOR are simply compounded interest rates that relate to the zero-coupon bond prices (discount factors) by

\[
\text{LIBOR}(X_t, \tau) = \frac{100}{\tau} \left( \frac{1}{P(X_t, \tau)} - 1 \right). \tag{20}
\]
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where the maturity $\tau$ follows actual-over-360 day-counting convention, starting two business days forward. The swap rates relate to the zero-coupon bond prices by

$$SWAP(X_t, \tau) = 100h \times \frac{1 - P(X_t, \tau)}{\sum_{i=1}^{h\tau} P(X_t, i/h)},$$

(21)

where $\tau$ denotes the maturity of the swap contract in years and $h$ denotes the number of payments in each year. The U.S. dollar swap contract makes two payments a year: $h = 2$.

In equations (20) and (21), we write the values of LIBOR and swap rates as functions of the state vector $X_t$. In the model with the deterministic jump component, the values of these interest rates also depend on the magnitude of the anticipated jump size at that date, $a_J(t)$.

We cast the estimation problem into a state space form, extract the distributions of the states at each date by using an efficient filtering technique, and then estimate the model using quasi maximum likelihood method, assuming normal pricing errors on LIBOR and swap rates. In this procedure, the dynamics of the interest rate factors $X_t$ determine the state propagation equation, which, in discrete time format, can be written as,

$$X_{t+1} = \Phi X_t + \sqrt{Q}\epsilon_{t+1},$$

(22)

where $\epsilon \sim IN(0, I)$, $\Phi = \exp(-\kappa \Delta t)$, and $Q = \sqrt{\Delta t} I$ with $\Delta t = 1/52$ being the discrete time (weekly) interval.

For the model with a deterministic jump component, the Gaussian feature of the state variable is unchanged, with the exception that we expect the last state variable $X_3$ to be shifted by a normally distributed random amount $a_J(t + \tau_J)$ three months down the road. Furthermore, we regard the time-$t$ conditional mean forecast of the jump size $a_J(t)$ as an additional state variable, which with $\tau_J$ fixed also evolves as a random walk as in equation (14). We augment a discrete version of equation (14) for the state propagation equation:

$$a_J(t+1) = a_J(t) + \sqrt{q_J \Delta \epsilon_{J,t+1}}.$$  

(23)

We define the measurement equations on the LIBOR and swap rates by assuming additive normal pricing errors,

$$y_t = \begin{bmatrix} LIBOR(X_t, i) \\ SWAP(X_t, j) \end{bmatrix} + e_t, \quad \text{cov}(e_t) = R, \quad i = 1, 2, 3, 6, 12 \text{ months} \quad j = 2, 3, 5, 7, 10, 15, 30 \text{ years}.$$  

(24)

When the state variables are Gaussian and the measurement equations are linear, the Kalman (1960) filter yields the efficient state updates in the least square sense. In our application, the state propagation in equation (22) is Gaussian linear, but the measurement equations are defined in terms of LIBOR and swap rates, which are
nonlinear functions of the state variables. The traditional literature approximates the nonlinear function by using Taylor expansion and obtains various extended forms of the Kalman Filter (EKF). Since the ultimate objective is to obtain the posterior distribution of the state variables given the observations, Julier and Uhlmann (1997) propose the unscented Kalman Filter (UKF) to directly approximate the posterior density using a set of deterministically chosen sample points (sigma points). These sample points completely capture the true mean and covariance of the Gaussian state variables. When these sigma points are propagated through the nonlinear functions of LIBOR and swap rates, they capture the posterior mean and covariance accurately to the second order for any nonlinearity. This approach is computationally efficient, because it avoids the calculation of derivatives for the linear approximation. The method is also more accurate than EKF, since it reduces the convexity bias induced in the first-order approximation in the EKF. Appendix presents the technical details on the estimation.

For the estimation, we assume that the pricing errors on each series are independent, but with distinct variance $\sigma^2$. We also constrain $\kappa$ and $\kappa^*$ to be lower triangular matrices. Altogether, we have 31 parameters for the model without jumps: $\Theta \equiv [\kappa \in \mathbb{R}^6, \kappa^* \in \mathbb{R}^6, b_r \in \mathbb{R}^{3+}, b_{v_j} \in \mathbb{R}^3, a_r \in \mathbb{R}, \sigma \in \mathbb{R}^{12+}]$. The model with jumps has two additional parameters: $\sigma^2_J$ and $q_J$.

5. The Impact of Anticipated Fed Policy Changes on the Term Structure

Based on the estimation results on models with and without an anticipated jump component, we study the impacts of the anticipated jump component on different aspects of the term structure, including the general model performance, the factor dynamics, the relative contribution of each factor to the term structure, and the mean term structure.

5.1 MODEL PERFORMANCE

We first analyze the model’s performance based on its ability to capture the observed term structure. Given model parameter estimates, the filtering technique directly yields a time series of ex post updates on the state variables. We compute the model values for the LIBOR and swap rates based on equations (20) and (21) and compare them to the market quotes.

Table I reports the sample properties of the pricing errors on each of the 12 series under both models. We define the pricing error, in basis points, as the difference between the market quotes and the model-implied values for the 12 LIBOR and
Table I. Summary statistics of pricing errors on the U.S. dollar LIBOR and swap rates

Entries report the summary statistics of the pricing errors on the U.S. dollar LIBOR and swap rates under the three-factor Gaussian affine model (left hand side under “Without Jump”), and its extended version with a deterministic jump component (right hand side under “With Jump”). The data consist of weekly observations on LIBOR at maturities of one, two, three, six, and 12 months, and swap rates at maturities of two, three, five, seven, ten, 15, and 30 years. For each series, the data are closing Wednesday mid-quotes from January 4, 1995 to July 11, 2007 (654 observations). We estimate the model by using quasi-maximum likelihood method joint with unscented Kalman filter. We define the pricing error as the difference between the observed interest rate quotes and the model-implied fair values, in basis points. The columns titled Mean, MAE, Std, Auto, and Max denote, respectively, the sample mean, the mean absolute error, the standard deviation, the first order autocorrelation, and the maximum in absolute magnitude of the pricing errors for each series. The last column (VR) reports the percentage variance explained for each series by the model factors. The last row reports average statistics.

<table>
<thead>
<tr>
<th>Model Maturity</th>
<th>Without Jump</th>
<th>With Jump</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>MAE</td>
</tr>
<tr>
<td>1 m</td>
<td>0.54</td>
<td>6.71</td>
</tr>
<tr>
<td>2 m</td>
<td>0.43</td>
<td>2.95</td>
</tr>
<tr>
<td>3 m</td>
<td>0.35</td>
<td>1.14</td>
</tr>
<tr>
<td>6 m</td>
<td>-2.65</td>
<td>6.19</td>
</tr>
<tr>
<td>1 y</td>
<td>-6.95</td>
<td>9.40</td>
</tr>
<tr>
<td>2 y</td>
<td>-0.64</td>
<td>3.01</td>
</tr>
<tr>
<td>3 y</td>
<td>0.24</td>
<td>0.76</td>
</tr>
<tr>
<td>5 y</td>
<td>0.12</td>
<td>1.63</td>
</tr>
<tr>
<td>7 y</td>
<td>-0.18</td>
<td>1.34</td>
</tr>
<tr>
<td>10 y</td>
<td>-0.19</td>
<td>0.84</td>
</tr>
<tr>
<td>15 y</td>
<td>1.65</td>
<td>3.72</td>
</tr>
<tr>
<td>30 y</td>
<td>-0.58</td>
<td>6.15</td>
</tr>
<tr>
<td>Average</td>
<td>-0.66</td>
<td>3.65</td>
</tr>
</tbody>
</table>

On average, incorporating a deterministic jump component in the factor dynamics reduces the mean absolute pricing error from 3.65 basis points to 2.63 basis points, reduces the standard deviation of the pricing error from 4.97 to 3.59 basis points, and reduces the remaining serial correlation in the pricing error from 0.78 to 0.63. The explained percentage variance (VR) improves from 99.81 to 99.9 percent. Inspecting the pricing error summary statistics on different interest rate series, we
find that incorporating the jump component reduces the pricing error not only on short-term LIBOR rates, but also on long-term swap rates.

The significance of the improvement also shows up in the log likelihood values of the two models, reported in the last column (titled “$L$”) under Table II. The log likelihood value for the model without a jump is 17,627 and that for the model with the anticipated jump component is 19,207, a difference of 1,579. Since the jump model nests the model without the jump component, we can construct a Chi-square test on the log likelihood ratio with a degree of freedom of 656 (654 weekly observations for the anticipated jump size plus two augmented parameters). The test generates a $p$-value that is not distinguishable from zero, indicating that the improvement is statistically significant over any reasonable confidence level.

Incorporating an anticipated jump component in the factor dynamics significantly improves the model performance both in matching the observed term structure of interest rates and in raising the log likelihoods of the observations. These findings contrast with the analysis of Johannes (2004), who shows that a random jump component has little impact on the term structure of interest rates. By making the arrival rate of the random jump strongly time and state-dependent, Piazzesi (2005) shows that the random jump component can have significant impacts on the term structure, particularly at the short end of the yield curve. When using a deterministic jump component, we show that the impact is not limited to the short-term interest rates, but extends to very long maturities.

5.2 FACTOR DYNAMICS

Based on the model parameter estimates, we analyze how the estimated factor dynamics differ in models with and without the anticipated jump component. In Table II, we report the model parameter estimates. We also report the absolute magnitudes of the $t$-values for each parameter in parentheses. The row titled “No Jump” contains parameter estimates for the model without the jump component. The row titled “With Jump” contains the estimates for the model with the anticipated jump component.

The parameter estimates on $\kappa$ control the mean-reverting feature of the time-series dynamics. For the factor dynamics to be stationary under the statistical measure, the real part of the eigenvalues of the $\kappa$ matrix must be positive. In the estimation, we constrain the $\kappa$ matrix to be lower triangular and the diagonal terms of the matrix to be nonnegative so that the factor dynamics maintain stationarity. The eigenvalues of the lower triangular matrix coincide with the diagonal elements of the matrix. Under both model specifications, the first two diagonal elements of the $\kappa$ matrix are not statistically different from zero, indicating that the first two interest rate factors are highly persistent. Nevertheless, using over 12 years of data, we can estimate the mean reversion speed of the third factor with statistical significance.
Table II. Parameter estimates for three-factor Gaussian affine models with and without anticipated jumps on the U.S. dollar term structure

Entries report the parameter estimates (and the absolute magnitude of the t-values in parentheses) of the three-factor Gaussian affine model (top panel) and its extended version with a deterministic jump component (bottom panel). We estimate the two models by using a quasi-maximum likelihood method joint with unscented Kalman filter, based on U.S. dollar LIBOR and swap rates (12 series). The LIBOR have maturities at one, two, three, six, and 12 months. Swap rates have maturities at two, three, five, seven, ten, 15, and 30 years. For each series, the data are closing Wednesday mid-quotes from January 4, 1995 to July 11, 2007 (654 observations).

<table>
<thead>
<tr>
<th>Model</th>
<th>κ</th>
<th>κ*</th>
<th>b_r</th>
<th>b_P</th>
<th>a_r</th>
<th>( \sigma^2_{QJ} )</th>
<th>( \mathcal{L} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Jump</td>
<td>0.079 (0.31)</td>
<td>0.014 (20.2)</td>
<td>0.000</td>
<td>-0.116</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-0.051 (0.17)</td>
<td>0.059 (2.3)</td>
<td>0.000</td>
<td>0.628</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.364 (0.61)</td>
<td>-2.957 (14.7)</td>
<td>0.0047</td>
<td>-7.754</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>With Jump</td>
<td>0.148 (0.50)</td>
<td>0.004 (8.8)</td>
<td>0.000</td>
<td>-0.156</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-0.292 (0.59)</td>
<td>0.081 (3.2)</td>
<td>0.000</td>
<td>0.709</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.342 (0.44)</td>
<td>-2.256 (15.7)</td>
<td>0.0054</td>
<td>-5.647</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\( \mathcal{L} \) values in parentheses.
The parameters in the $\kappa^*$ matrix control the factor dynamics under the risk-neutral measure. The risk-neutral dynamics determine the cross-sectional behavior of interest rates at different maturities, hence the term structure. Unlike the low $t$-values for the $\kappa$ estimates, the estimates for $\kappa^*$ are strongly significant in both models. Comparing the estimates for the two models, we find that the three diagonal elements of $\kappa^*$ are all smaller under the model with the jump component than under the model without the jump component. The smaller diagonal elements under the jump model suggest that the factors are more persistent under the risk-neutral measure and therefore have a larger impact on interest rates of longer maturities.

To see how the risk-neutral factor dynamics affect the term structure of interest rates, we note that the continuously compounded spot rate at maturity $\tau$ is affine in the state vector, 

$$y(X_t, \tau) = \left[ \frac{a(\tau)}{\tau} \right] + \left[ \frac{b(\tau)}{\tau} \right]^\top X_t. \quad (25)$$

The slope coefficient $b(\tau)/\tau$ defines the relative contribution of each factor to the spot rate at maturity $\tau$, which we label as factor loading. We can think of the factor loading as the instantaneous response of the spot rate on the three different shocks (factors). According to the ordinary differential equations in equation (7), the factor loading is determined by the loading of the factors to the short rate ($b_r$) and the risk-neutral factor dynamics ($\kappa^*$). As shown in Table II for both models, only the estimate for the last element of $b_r$ is visually different from zero. Hence, the short rate is mainly controlled by the third factor. As the maturity increases, the risk-neutral factor dynamics $\kappa^*$ interacts with the short-rate loading $b_r$ to determine the factor loading on the long-term rates. The two panels in Figure 1 plot the factor loadings on the continuously compounded spot rates under the two models, based on the parameter estimates in Table II. The left panel depicts the loading under the three-factor Gaussian affine model without jumps. The right panel is the loading under the model with an anticipated jump component.

Under both models, the first, and also the most persistent, factor (solid lines) influences mostly the long-term interest rates. The third, and the most transient, factor (dash-dotted lines) influences mostly the short-term rates. The second factor’s loading (dashed lines) reaches a maximum at some moderate maturity.

More interesting to us is how incorporating an anticipated jump component alters the factor loading by altering the estimated risk-neutral factor dynamics. The most obvious difference between the two panels in Figure 1 is the loading of the first factor. In the model with the anticipated jump component (right panel), the loading of the first factor increases monotonically with maturity and flattens out after ten years. In contrast, without the jump component (left panel), the loading of this first factor reaches a plateau around five years. The loading actually declines as maturity...
increases further. Thus, incorporating the jump component shifts the loading of the first factor to longer-term rates. A similar observation also applies to the second factor.

The more persistent factors dominate the behavior of longer-term interest rates. In the absence of the anticipated jump component, the first and second factors are forced to play some roles in the short-term interest rates, accommodating the distorted behavior of the short-term interest rates that anticipate the target federal funds rate moves. When we add an anticipated jump component to the transient third factor, this jump component accommodates the market anticipation of the Federal Reserve policy changes and captures the short-end distortions of the yield curve that are due to this anticipation. As a result, the two persistent factors are freed up to capture the long-term interest-rate behavior. This interaction in part explains why incorporating an anticipated jump component in the short-term rates (the most transient factor) improves the model’s performance in capturing long-term swap rates.

5.3 THE IMPACTS OF AN ANTICIPATED 100-BPS SHORT-RATE HIKE

To illustrate the impact of an anticipated jump, we propose an anticipated short rate hike (positive jump) by 100 basis points three months later. The impact of an anticipated jump on the mean short rate is given by $b_t(3)\bar{\alpha}_J$. Given the estimate for $b_t(3) = 0.0054$, a 100-basis-point anticipated short-rate jump implies an anticipated jump on the third factor by $\bar{\alpha}_J = 1.853$.

In the left panel of Figure 2, we illustrate the impact of such an anticipated jump on the average term structure of the spot interest rates. The dashed line plots...
Figure 2. The impact of an anticipated 100-basis-point short-rate hike. The solid lines denote the mean spot rate curve (left panel) and mean swap rate curve (right panel) in anticipation of a 100-basis-point hike in the short rate three month later. The dash-dotted lines denote the benchmark case in the absence of such anticipation. We compute the mean spot rates and swap rates based on the estimated three-factor Gaussian affine model with an anticipated jump component for the U.S. dollar term structure.

Given the average spot-rate curve, we can also compute the mean swap-rate curve according to equation (21). The right-hand panel of Figure 2 plots the mean swap rate curve in anticipation of the 100-basis-point short-rate hike (solid line). The dashed line again represents the benchmark without any anticipated jumps. The largest impact of the anticipated hike on swap rates comes at around one year maturity for 51.6 basis points, but the impact also reaches the longest maturities. The impact on the 30-year swap rate is 6.54 basis points. Thus, incorporating an
5.4 FEDERAL FUNDS RATE TARGET CHANGES: ANTICIPATION VERSUS REALIZATION

From the estimated model with the anticipated jump component, we can extract a time series of anticipated jump sizes $\alpha_J(t)$ as the fourth element of the state vector that best explains the current yield curve. We motivate this jump specification based on the discrete and well-anticipated changes in the federal funds target rate. In this section, we analyze how the anticipated mean jump size that we extract from the LIBOR and swap market relates to the actual target changes on the federal funds rate.

Given the anticipated jump size $\alpha_J(t)$ at each date $t$, the anticipated move on the short rate is given by $b(t)\alpha_J(t)$. Figure 3 plots the time series of this anticipated short-rate jump as the dashed line. To compare this anticipated jump with the actually realized federal funds target rate changes, we obtain the history of the federal funds target rates from Bloomberg and calculate the corresponding actual aggregate target rate changes over the next three months at each point in time. The time series is plotted as the solid line in Figure 3. During our sample period, the federal funds target rates changed 44 times, in either 25 or 50 basis points. Comparing the anticipated short-rate jumps with the target rate changes, we observe that during our 12-year sample period, the market anticipates the federal funds rate
target move very well. This market anticipation shows up vividly on the interest-rate term structure. This observation implies that during this sample period, the Federal Reserve rarely surprises or goes against the market in making federal funds rate target changes.

Nevertheless, during our sample period, the market seems to have a downward bias in anticipating the Federal Reserve moves. The sample average of the anticipated short rate jump is $-21.66$ basis points, whereas the sample average of the actual three-month aggregate federal funds target rate changes is $-0.99$ basis points. There are several reasons that can generate such a bias. First, our model uses only one jump with a fixed time horizon of three months to summarize the impacts of potential future target rate changes, but in reality target rate changes of the same direction can also happen after three months. Second, since $\bar{a}_j(t)$ is the conditional mean of the random jump under the risk-neutral measure, its average deviation from the sample average of the actual target rate changes can represent a risk premium component: The Federal Reserve has been more concerned with fighting inflation and implementing a tighter monetary policy than the market has wanted. The market’s wish for more rate cuts can also be related to the so-called “Greenspan put” phenomenon (Miller, Weller, and Zhang (2001)): Investors in the United States expect that the Federal Reserve will take decisive actions to prevent the market from falling, but will not stop it from rising, thus providing a put-option-like insurance on the market.


In previous sections, we use one anticipated jump to summarize the impact of Fed policy changes. In reality, the Fed policy change is not a random event, but depends crucially on the state of the economy. Furthermore, when the economic conditions ask for large rate changes, the Federal Reserve often implements the policy change gradually through a series of consecutive small-sized target rate moves. Empirically, Balduzzi, Bertola, and Foresi (1997) find that the Fed target movements are strongly serially correlated. Bernanke (2004b) label this phenomenon as “gradualism.”

In this section, we consider an alternative jump specification that moves closer to the above features of the federal funds target rate. To model the gradualism, we relax the one jump assumption and allow a consecutive jumps of equal distance that are serially correlated. Let $\tau_J$ denote the time interval between adjacent jumps, we specify the following autoregressive structure (under the risk-neutral measure $\mathbb{P}^*$)

---

4 Many recent studies link the monetary policy rule on the short rate to systematic economic conditions such as expected inflation and output growth. Examples include Hordahl, Tristani, and Vestin (2006), Rudebusch and Wu (2008), Ang, Dong, and Piazzesi (2004), Bekaert, Cho, and Moreno (2005), Bikbov and Chernov (2005), and Gallmeyer, Hollifield, and Zin (2005).
between the random sizes of adjacent jumps,

\[ a(t + j \tau J) = \varphi_J a(t + (j - 1) \tau J) + e_j, \quad j > 1, \]  

(28)

where \( \varphi_J \in (0, 1) \) captures the serial dependence structure of the consecutive jumps and the noise term \( e_j \) is assumed to be independent, identical, and normally distributed with zero mean and variance \( V_e \). To achieve the separation of expectation operations analogous to (13), we maintain the independent assumption between each jump and the pre-jump state vector level, but we allow the conditional forecast of the mean jump size to be a function of the current state of the economy and past Fed policy changes. Specifically, we assume that conditional on time-\( t \) filtration \( \mathcal{F}_t \) and under the risk-neutral measure \( \mathbb{P}^* \), the random jump size of the nearest jump \( (j = 1) \) with a horizon of \( \tau J \) is normally distributed,

\[ a_J (t + \tau J) \sim N \left( \varphi_J a_J (t - \tau J) + b_J^T X_t + \bar{a}_J(t), \sigma_J^2 \right). \]  

(29)

The conditional mean jump size has three components: (1) the most recent federal funds target rate change, which we scale by \( b_J \) to obtain the last realized jump on the third factor denoted by \( a_J (t - \tau J) \), (2) the current state of the economy captured by the state vector \( X_t \), and (3) an independent source of market anticipation \( \bar{a}_J(t) \) not captured by the other variables. We retain the random walk updating assumption on \( \bar{a}_J(t) \) as in (14). As shown in Section 3.2, the effects of the updating dynamics and the market pricing of the updating risk on the mean and variance of the distribution can be absorbed in the specified \( \bar{a}(t) \) and \( \sigma_J^2 \) levels.

Combining the assumptions in (28) and (29), we can derive the mean and covariance matrix of future random jump sizes, \( A_J = [a(t + \tau J), a(t + 2 \tau J), \ldots]^T \). The time-\( t \) conditional mean forecast on the jump vector is \( \bar{X}_J(t) = (\varphi_J a_J (t - \tau J) + b_J^T X_t + \bar{a}_J(t)) \Psi_a \), with \( \Psi_a = [1, \varphi_J, \ldots]^T \) denoting the serial dependence vector with its \( i \)-th element given by \( \varphi_J^{(i-1)} \). The covariance matrix \( V_A \) is composed of the following diagonal and off-diagonal elements,

\[ [V_A]_{ii} = \varphi_J^{2(i-1)} \sigma_J^2 + \left( 1 + \cdots + \varphi_J^{2(i-2)} \right) V_e, \quad \text{and} \quad [V_A]_{ij} = \varphi_J^{(i-j)} \sigma_J^2, \quad \text{for } i \neq j. \]  

(30)

The bond pricing has a separation of expectations on the pre-jump state vector and the random jumps analogous to equation (13),

\[ P(X_t, \tau) = \left[ \exp(-a (\tau) - b (\tau)^T X_t) \right] \left[ \mathbb{E}^* \left( e^{−B_J(t)^T A_J} \right) \right], \]  

(31)

where \( B_J(t) = [b (\tau - \tau J), b (\tau - 2 \tau J), \ldots]^T \) denotes the loading vector on the vector of jumps \( A_J \). The loading coefficient for the \( i \)-th jump is zero if \( \tau - i \tau J < 0 \). Therefore, if we use \( N_J(\tau) \) to denote the integer component of \( \tau / \tau J \), the bond price with maturity \( \tau \), \( P(X_t, \tau) \), is affected only by the first \( N_J(\tau) \) jumps. As a result, we
can think of $B_J(\tau)$ and $A_J$ in equation (31) as $N_J(\tau) \times 1$ vectors. The impacts of these $N_J(\tau)$ jumps are captured by the following expectation operation,

$$\mathbb{E}^* \left( e^{-B_J(\tau)^\top X(t)} \right) = e^{-B_J(\tau)^\top \Psi_A(\psi_JA_J(t_-)+\bar{\sigma}_J(t)) + \frac{1}{2} B_J(\tau)^\top V_A B_J(\tau) - (B_J(\tau)^\top \Psi_A)\phi_{J a_J}(t_-) + (a_J(t_-)) + \frac{1}{2} B_J(\tau)^\top V_A B_J(\tau)^\top (B_J(\tau)^\top \Psi_A)\phi_{J a_J}(t_-)},$$

which remains exponential affine in the state vector $X_t$. The linkage between the affine coefficients with and without the jumps becomes,

$$a^{t}(\tau, t) = a(\tau) + B_J(\tau)^\top \Psi_A(\psi_Ja_J(t_-) + \bar{\sigma}_J(t)) - \frac{1}{2} B_J(\tau)^\top V_A B_J(\tau),$$

$$b^{t}(\tau) = b(\tau) + (B_J(\tau)^\top \Psi_A) b_J.$$

Since the conditional mean jump sizes depend linearly on the state vector $X_t$, the presence of jumps now also affects the linear coefficients $b^{t}(\tau)$. Alternatively, if we represent the bond pricing equation in terms of the expanded state vector $Z_t = [X_t^\top, \bar{\sigma}_J(t)]^\top$ as in (18), the affine coefficients become,

$$c(\tau, t) = a(\tau) + B_J(\tau)^\top \Psi_A(\psi_Ja_J(t_-) + \bar{\sigma}_J(t)) - \frac{1}{2} B_J(\tau)^\top V_A B_J(\tau),$$

$$d(\tau)_{(1:3)} = b(\tau) + (B_J(\tau)^\top \Psi_A) b_J,$$

$$d(\tau)_{4} = B_J(\tau)^\top \Psi_A.$$

In this case, the intercept coefficient $c(\tau, t)$ also depends on the past policy change $a(t_-)$, which is not an additional state variable, but an exogenously observable quantity.

We estimate the model with this alternative jump specification on the same data set. With multiple jumps allowed, we set the equal-distant jump interval $\tau_J$ to be smaller at one month.\(^5\) Table III reports the parameter estimates, the $t$-statistics, and the maximized log likelihood value. The log likelihood value of this alternative jump specification is much greater than that from the original jump specification.

To test the statistical significance of the performance difference between the two models, we construct a likelihood ratio test for the two non-nested models according to Vuong (1989),

$$M = (L_2 - L_1 \sqrt{N}),$$

where $L_1$ and $L_2$ denote the maximized aggregate log likelihood values for the two models, $\hat{s}$ denotes the variance estimate of the weekly log likelihood ratio $(l_2 - l_1)$, and $N$ denotes the number of weekly observations for each series. Vuong proves that $M$ has an asymptotic standard normal distribution under the null hypothesis that the two models are equivalent in terms of likelihood. Based on the weekly log likelihood estimates for the two models, we compute the sample mean and standard deviation of the likelihood ratio.

\(^5\) We have also estimated an analogous model with the jump interval $\tau_J$ set to be three months. The model’s performance is not as good as the one-month jump interval specification but is significantly better than the original one-jump specification.
Entries report the parameter estimates (and the absolute magnitude of the \( t \)-values in parentheses) of the three-factor Gaussian affine model with serially correlated jumps with the mean jump size dependent on the current state of the economy. We estimate the model by using a quasi-maximum likelihood method joint with unscented Kalman filter, based on U.S. dollar LIBOR and swap rates (12 series). The LIBOR have maturities at one, two, three, six, and 12 months. Swap rates have maturities at two, three, seven, ten, 15, and 30 years. For each series, the data are closing Wednesday mid-quotes from January 4, 1995 to July 11, 2007 (654 observations).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>t-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \kappa )</td>
<td>0.159 (0.68)</td>
<td>-</td>
</tr>
<tr>
<td>( \kappa^* )</td>
<td>0.213 (0.67)</td>
<td>(23.8)</td>
</tr>
<tr>
<td>( b_r )</td>
<td>0.016 (0.81)</td>
<td>(8.62)</td>
</tr>
<tr>
<td>( b_J )</td>
<td>0.010 (0.151)</td>
<td>(23.8)</td>
</tr>
<tr>
<td>( a_r )</td>
<td>0.000 (0.03)</td>
<td>(8.62)</td>
</tr>
<tr>
<td>( \sigma_J )</td>
<td>0.060 (0.71)</td>
<td>(23.8)</td>
</tr>
<tr>
<td>( V_c )</td>
<td>0.0054 (0.00)</td>
<td>(8.62)</td>
</tr>
<tr>
<td>( b_J )</td>
<td>0.016 (0.00)</td>
<td>(8.62)</td>
</tr>
<tr>
<td>( \psi_J )</td>
<td>0.016 (0.00)</td>
<td>(8.62)</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>20.076</td>
<td>(8.62)</td>
</tr>
</tbody>
</table>

Table III. A model with serially correlated jumps with the mean jump size dependent on the current state of the economy.
Figure 4. Anticipated and actual federal funds target rate changes over the next month. The dashed line plots the time series of the anticipated short-rate jump $b_r(3)(\varphi J a_J(t-\tau) + b_J^\top X_t + \pi_J(t))$ over the next month. The solid line plots the corresponding actual federal funds target rate change over the same time horizon. The anticipated short-rate jump is extracted from the U.S. dollar LIBOR and swap rate data based on the estimated three-factor Gaussian affine model with serially correlated random jumps with the mean jump size dependent on the current state of the economy.

In computing $\hat{\tau}$, we adjust the serial dependence of the likelihood ratio according to Newey and West (1987) with the lags optimally chosen following Andrews (1991) under an AR(1) specification. We obtain an estimate for the test statistic $M$ at 5.36, strongly rejecting the original specification in favor of the alternative jump specification. Therefore, allowing multiple jumps that are serially correlated and with mean jump sizes dependent on the current state of the economy significantly improves the model performance.

Of the parameter estimates, the most interesting to us are the estimates on the jump’ serial dependence ($\varphi_J$) and its dependence on the state of the economy ($b_J$). The estimated serial dependence at $\varphi_J = 0.88$ is high and strongly significant, consistent with the evidence in Balduzzi, Bertola, and Foresi (1997) and Bernanke’s argument for gradualism. The coefficient estimates on the three elements of the state vector are all negative. Since the yield curve have positive loadings on all three factors, the negative estimates on $b_J$ suggest that the market expects the Fed to cut the target rate when interest rates are high and raise the target rate when interest rates are low. Thus, the jump component introduces an additional mechanism for maintaining stationarity on interest rates.

Given the parameter estimates and extracted states from the unscented Kalman filter, we can compute the anticipated short rate jump size at each time $t$ as $b_r(3)(\varphi_J a_J(t-\tau) + b_J^\top X_t + \pi_J(t))$. Figure 4 compares the time series of this anticipated jump (dashed line) with the actual federal funds target rate changes (solid
line) over the same horizon. With a monthly horizon, both the anticipated and realized target rate changes are smaller in absolute magnitude compared to the plots in Figure 3. The incorporation of the serially correlated multiple jump structure further reduces the magnitude of the anticipated jump size. Nevertheless, the mean downward bias remains in the anticipated jump size as compared to the actual realized target rate changes. The sample average of the monthly anticipated short rate jump is $-11.14$ basis points, but the sample average of the monthly realized federal fund target rate change is only $-0.15$ basis points. Given the accommodation of multiple jumps over the whole horizon of the LIBOR/swap rates, the mean bias is more likely to reflect a risk premium on the random target rate changes.

7. Conclusion

The Federal Reserve implements its monetary policy mainly through adjustments to the target federal funds rate. These adjustments are discrete in time and result in discontinuity in short-term interest rates. To date, the literature has modeled these discontinuous moves using random Poisson jumps. Yet, random jumps only capture the Fed policy surprises, which have little impact on the yield curve, unless the arrival rates are made strongly time- and state-dependent.

In this paper, we propose a dynamic term-structure model with an anticipated jump component that captures the anticipated federal funds rate changes. Our estimation results show that incorporating such an anticipated jump component significantly improves the model's performance in pricing the interest rate term structure and predicting future interest rate movements. Further specification analysis on the jump size distribution shows that market anticipation of the Fed policy changes is not only related to past policy changes, but also related to the current state of the economy.

Understanding and forecasting Federal Reserve policy changes constitute an important topic in financial economics. By estimating the model to the panel data of LIBOR and swap rates, we extract a time series of anticipated federal funds rate moves that are consistent with the yield curve. Comparing this implied federal funds rate movements to the actual federal funds rate target changes, we find that historically, the yield curve, and hence the interest-rate market, anticipate the Fed moves well. The Fed has rarely surprised or gone against the market in making policy moves during the past decade. The Federal Reserve has been very active in both its target rate adjustments and its communications with the public. The frequent adjustment necessitates a jump component in term structure modeling. The increased transparency in policy making reduces the number of surprises to the market. The two elements together contribute to the success of our anticipated jump model.
There also exists a related strand of literature that tries to link macroeconomic variables to the federal funds rate adjustment. These studies serve two purposes. First, they help us better understand the link between macroeconomic variables and the federal reserve’s monetary policy. Second, one can use the identified relation to predict future monetary policy changes based on the current macroeconomic signals. Nevertheless, the relation between many macroeconomic variables and the federal funds rate changes is neither linear nor stable over time. Therefore, any statistical fit based on historical data may not be successful in predicting the future. In contrast, our analysis shows that the current yield curve is an efficient filter for the current information, macroeconomic and otherwise, and provides an accurate forecast for future federal funds rate changes.

Johannes (2004) shows that a random Poisson jump has little impact on the term structure but has important impacts on option pricing, because the random jump alters the distribution of future interest rates. Since the evidence indicates that most of the federal funds rate target changes are well anticipated by the market, it is important to understand the different roles played by policy surprises and anticipated moves. It is also important to empirically gauge the relative contribution of the two components to option pricing at different maturities and moneyness. We leave these issues for future research.

Appendix. Unscented Kalman filter and quasi-maximum likelihood

Kalman Filter and its various extensions belong to the state space estimation regime and are based on a pair of state propagation equations and measurement equations. In our application, the state vector $X$ propagates according to the Ornstein-Uhlenbeck process as described in equation (2). Its discrete-time version is,

$$X_t = \Phi X_{t-1} + \sqrt{Q}\epsilon_t,$$

(A1)

which is linear and conditionally normal. The measurement equation, as described in equation (24), is based on the observations on LIBOR and swap rates. We can rewrite the measurement equation in a generic form,

$$y_t = h (X_t; \Theta) + e_t,$$

(A2)

where $y_t$ here denotes the observed series at time $t$ and $h (X_t; \Theta)$ denotes their corresponding fair values based on the term structure model, as a function of the state vector $X_t$ and model parameters $\Theta$. The last term $e_t$ denotes the pricing error on the series at time $t$. We assume that the pricing error is independent of the state vector and that the pricing error on each series is also mutually independent, but with distinct variance $R_{ii} = \sigma_i^2, i = 1, \ldots, 24; R_{ij} = 0$ for $i \neq j$.

Let $\bar{X}_t$ and $\bar{y}_t$ denote the time-$(t - 1)$ ex ante forecasts of time-$t$ values of the state vector and the measurement series, let $\bar{X}_{XX,t}, \bar{X}_{XY,t}, \bar{Y}_{YY,t}$ denote the
corresponding forecasts on the variance and covariance matrices, and let \( \hat{X}_t \) and \( \hat{\Sigma}_{XX,t} \) denote the ex post update, or filtering, on the state vector and its covariance at the time \( t \) based on observations \( (y_t) \) at time \( t \). In the case of linear measurement equations,

\[
y_t = H X_t + e_t, \tag{A3}
\]
the Kalman filter provides the most efficient updates. The ex ante predictions are,

\[
\bar{X}_t = \Phi \hat{X}_{t-1}, \quad \bar{\Sigma}_{XX,t} = \Phi \hat{\Sigma}_{XX,t-1} \Phi^T + Q; \tag{A4}
\]

\[
\bar{y}_t = H \bar{X}_t, \quad \bar{\Sigma}_{yy,t} = H \bar{\Sigma}_{XX,t} H^T + R, \quad \bar{\Sigma}_{XY,t} = \bar{\Sigma}_{XX,t} H^T. \tag{A5}
\]

The ex post filtering updates are,

\[
\hat{X}_t = \bar{X}_t + K_t (y_t - \bar{y}_t), \quad \hat{\Sigma}_{XX,t} = \bar{\Sigma}_{XX,t} - K_t \bar{\Sigma}_{YY,t} K_t^T, \tag{A6}
\]

where \( K_t = \bar{\Sigma}_{XY,t}(\bar{\Sigma}_{YY,t})^{-1} \) is the Kalman gain.

In our application, the measurement equation in (A2) is nonlinear. Traditionally, nonlinearity is often handled by the Extended Kalman Filter (EKF), which approximates the nonlinear measurement equation with a linear expansion, evaluated at the predicted states:

\[
y_t \approx H(\bar{X}_t; \Theta) X_t + e_t, \tag{A7}
\]

where

\[
H(\bar{X}_t; \Theta) = \frac{\partial h(\bar{X}_t; \Theta)}{\partial X_t} \bigg|_{X_t=\bar{X}_t}. \tag{A8}
\]

The rest of prediction and updates follow equations (A4) to (A6). The extended Kalman filter uses only one point (the conditional mean) from the prior filtering density for the prediction and filtering updates.

In contrast, the unscented Kalman filter applied in this paper uses a set of points that are designed to also match higher moments. Let \( p \) be the number of states and \( \delta > 0 \) be a control parameter. Let \( A_i \) be the \( i \)th column of a matrix \( A \). A set of \( 2p+1 \) sigma vectors \( \chi_i \) are generated according to the following equations:

\[
\chi_{i,0} = \bar{X}_t, \quad \chi_{i,j} = \bar{X}_t \pm \sqrt{(p+\delta)(\bar{\Sigma}_{XX,t})}, \quad j = 1, \ldots, p; \quad i = 1, \ldots, 2p, \tag{A9}
\]

with corresponding weights \( w_i \) given by

\[
w_0 = \delta/(p + \delta), \quad w_i = 1/[2(p + \delta)], \quad j = 1, \ldots, 2p. \tag{A10}
\]

We can regard these sigma vectors as forming a discrete distribution with \( w_i \) being the corresponding probabilities. We can verify that the mean and covariance of this distribution are \( \bar{X}_t \) and \( \bar{\Sigma}_{XX,t} \), respectively.
Given the Gaussian-linear structure of the state propagation equation, we can still use equation (A4) to predict the mean and covariance of the state vector $X_t$ and $\Sigma_{XX,t}$. Then, we generate the sigma points based on the predicted mean and covariance according to equations (A9) and (A10), and use these sigma points to predict the mean and covariances of the measurement series,

$$
\begin{align*}
\overline{y}_t &= \sum_{i=0}^{2p} w_i h(\chi_{t,i}; \Theta), \\
\Sigma_{yy,t} &= \sum_{i=0}^{2p} w_i [h(\chi_{t,i}; \Theta) - \overline{y}_t][h(\chi_{t,i}; \Theta) - \overline{y}_t]^T + R, \\
\overline{\Sigma}_{Xy,t} &= \sum_{i=0}^{2p} w_i [\chi_{t,i} - \overline{X}_t][h(\chi_{t,i}; \Theta) - \overline{y}_t]^T.
\end{align*}
$$

(A11)

With these predicted moments, the filtering follows the same steps as in equation (A6).

To estimate the model parameters, we define the log-likelihood for each day’s observation assuming that the forecasting errors are normally distributed:

$$
l_t(\Theta) = -\frac{1}{2} \log |A_t| - \frac{1}{2} ((y_t - \overline{y}_t)^T (A_t)^{-1} (y_t - \overline{y}_t)).
$$

(A12)

We choose model parameters to maximize the log likelihood of the data series, which is a summation of the daily log likelihood values,

$$
\Theta \equiv \arg \max_{\Theta} \mathcal{L} \left( \Theta, \{y_t\}_{t=1}^{N} \right), \quad \text{with} \quad \mathcal{L} \left( \Theta, \{y_t\}_{t=1}^{N} \right) = \sum_{t=1}^{N} l_t(\Theta),
$$

(A13)

where $N$ denotes number of days in our sample.

References


