Managing the Risk of Options Positions

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When to take option positions?

1. Increase leverage, & increase exposures to the underlying while mitigating information leakage and market impact
   - Positioning in the underlying instead of derivatives.

2. Hedge the tail risk of the underlying positions
   - Used as insurance to hedge against crashes of the market, defaults of single names ...

3. Gain exposures and receive risk premiums on higher-order risks (variance risk, credit risk, crash risk)
   - The risk premium tends to be large but the risk tends to be damaging.

4. Statistical arbitrage trading (based on no-arbitrage models)
   - Trading against relative mispricings of options contracts while minimizing the risk exposures of the portfolios

5. Options market making
   - Receiving spreads while minimizing the risk of the derivatives inventory

My discussion is mostly applicable to the last two applications.
Review: Valuation and investment in primary securities such as stocks, bonds

- The securities have direct claims to future cash flows.
- Valuation is based on forecasts of future cash flows and risk:
  - DCF (Discounted Cash Flow Method): Discount time-series forecasted future cash flow with a discount rate that is commensurate with the forecasted risk.
- Investment:
  - “Alpha” can be defined as the difference between value and price. Buy if market price is lower than model value; sell otherwise.
  - Value and invest in as many (different) securities as practically possible to increase the width of the investment.
  - Covariance matrix construction and risk factor analysis are crucial in forming balanced portfolios and in managing portfolio risks.
- Both valuation and investment depend crucially on forecasts of future cash flows (growth rates) and risks (beta, credit risk).
Payoffs are linked directly to the price of an “underlying” security.

Valuation is mostly based on replication/hedging arguments.

- Find a portfolio that includes the underlying security, and possibly other related derivatives, to replicate the payoff of the target derivative security, or to hedge away the risk in the derivative payoff.
- Since the hedged portfolio is riskfree, the payoff of the portfolio can be discounted by the riskfree rate.
- Models of this type are called “no-arbitrage” models.

Key: No forecasts are involved. Valuation is based on cross-sectional comparison.

- It is not about whether the underlying security price will go up or down (forecasts of growth rates /risks), but about the relative pricing relation between the underlying and the derivatives under all possible scenarios.
Readings behind the technical jargons: $\mathbb{P}$ v. $\mathbb{Q}$

- **$\mathbb{P}$**: Actual probabilities that earnings will be high or low, estimated based on historical data and other insights about the company.
  
  Valuation is all about getting the forecasts right and assigning the appropriate price for the forecasted risk — *fair wrt future cashflows (and your risk preference)*.

- **$\mathbb{Q}$**: “Risk-neutral” probabilities that one can use to aggregate expected future payoffs and discount them back with riskfree rate.
  
  It is related to real-time scenarios, but it is not real-time probability.
  
  Since the intention is to *hedge* away risk under all scenarios, one does not really care about the actual probability of each scenario happening. One just cares about what all the possible scenarios are and whether the hedging works under *all* scenarios.
  
  $\mathbb{Q}$ is not about getting close to the actual probability, but about being *fair/consistent wrt the prices of securities that you use for hedging*.

- Associate $\mathbb{P}$ with *time-series* forecasts and $\mathbb{Q}$ with *cross-sectional* comparison.
A Micky Mouse example

Consider a non-dividend paying stock in a world with zero riskfree interest rate (zero financing cost). Currently, the market price for the stock is $100. What should be the forward price for the stock with one year maturity?

- The forward price is $100.
  - Standard forward pricing argument says that the forward price should be equal to the cost of buying the stock and carrying it over to maturity.
  - The buying cost is $100, with no storage or interest cost or dividend benefit.

- How should you value the forward differently if you have inside information that the company will be bought tomorrow and the stock price is going to double?
  - Shorting a forward at $100 is still safe for you if you can buy the stock at $100 to hedge.
  - The concern is less about what the stock price will be, but about at what price you can buy/sell the stock (and for how much) before it moves away.
Investing in derivative securities without insights

- If you can really forecast (or have inside information on) future cashflows, you probably do not care much about hedging or no-arbitrage pricing.
  - What is risk to the market is an opportunity for you.
  - You just lift the market and try not getting caught for insider trading.

- If you do not have insights on cash flows and still want to invest in derivatives, the focus does not need to be on forecasting, but on cross-sectional consistency.
  - No-arbitrage option pricing models can be useful, in identifying cross-sectional inconsistencies.
  - Identifying the right hedge strategy can be equally (if not more) important, in managing the risk of the options portfolio.
    - In certain applications such as market making, risk management is probably more important than valuation.
What are the roles of an option pricing model?

Interpolation and extrapolation:

- Broker-dealers: Calibrate the model to actively traded option contracts, use the calibrated model to generate option values for contracts without reliable quotes (for quoting or book marking).

- Criterion for a good model: The model value needs to match observed prices (small pricing errors) because market makers cannot take big views and have to passively take order flows.

- Sometimes one can get away with interpolating (linear or spline) without a model.

- The need for a **cross-sectionally consistent** model becomes stronger for extrapolation from short-dated options to long-dated contracts, or for interpolation of markets with sparse quotes.

- The purpose is less about finding a better price or getting a better prediction of future movements, but more about making sure the provided quotes on different contracts are “consistent” in the sense that no one can buy and sell with you to lock in a profit on you.
What are the roles of an option pricing model?

2 Alpha generation

- Investors: Hope that market price deviations from the model “fair value” are mean reverting.
- Criterion for a good model is not to generate small pricing errors, but to generate (hopefully) large, transient errors.
- When the market price deviates from the model value, one hopes that the market price reverts back to the model value quickly.
- One can use an error-correction type specification to test the performance of different models.
- However, alpha is only a small part of the equation, especially for investments in derivatives.
- Execution (how to execute to reduce transaction cost or receive spreads) and risk management (making sure that the portfolio is truly close to “riskfree”) are equally important.
What are the roles of an option pricing model?

3 Risk hedge and management:

- Hedging and managing risk plays an important role in derivatives.
- Market makers make money by receiving bid-ask spreads, but options order flow is so sparse that they cannot get in and out of contracts easily and often have to hold their positions to expiration.
  - Without proper hedge, market value fluctuation can easily dominate the spread they make.
- Investors (hedge funds) can make active derivative investments based on the model-generated alpha. However, the convergence movement from market to model can be a lot smaller than the market movement, if the positions are left unhedged. — Try the forward example earlier.
  - Think of market making and statistical arbitrage as “picking pennies in front of a steamroller.”
- The estimated model dynamics provide insights on how to manage/hedge the risk exposures of the derivative positions.
- Criterion of a good model: The modeled risks are real risks and are the only risk sources.
- Once one hedges away these risks, no other risk is left.
What are the roles of an option pricing model?

Answering economic questions with the help of options observations

- Macro/corporate policies interact with investor preferences and expectations to determine financial security prices.
- Reverse engineering: Learn about policies, expectations, and preferences from security prices.
  - At a fixed point in time, one can learn a lot more from the large amount of option prices than from a single price on the underlying.
  - Long time series are hard to come by. The large cross sections of derivatives provide a partial replacement.
- The Breeden and Litzenberger (1978) result allows one to infer the risk-neutral return distribution from option prices across all strikes at a fixed maturity. Obtaining further insights often necessitates more structures/assumptions/an option pricing model.
- Criterion for a good model: The model contains intuitive, easily interpretable, economic meanings, and extracts a few useful economic indicators from the large amount of options data.
Forward & futures: A one-page review

- **Terminal Payoff**: \((S_T - K)\), linear in the underlying security price \((S_T)\).

- **No-arbitrage pricing** (replicating):
  - Buying the underlying security and carrying it over to expiry generates the same payoff as signing a forward.
  - The forward price should be equal to the cost of the replicating portfolio (buy & carry).
  - This pricing does not depend on (forecasts) how the underlying security price moves in the future or whether its current price is fair.
  - but it does depend on the actual cost of buying and carrying (not all things can be carried through time...), i.e., the implementability of the replicating strategy.

- When buying&carrying is difficult (e.g., electricity), the price can contain forecasts.
Options

- **Terminal Payoff**: Call — \((S_T - K)^+\), put — \((K - S_T)^+\).
  European, American, ITM, OTM, ATMV...

- **No-arbitrage pricing**:
  - Can we replicate the payoff of an option with the underlying security so that we can value the option as the cost of the replicating portfolio?
    - It is hard to replicate a link with linear lines.
  - Can we use the underlying security to completely hedge away the risk?
    - Under some seemingly extreme assumptions.
Another Mickey Mouse example: A one-step binomial tree

- **Observation**: The current stock price \( S_t \) is $20. The current continuously compounded interest rate \( r \) (at 3 month maturity) is 12%. (crazy number from Hull’s book).

- **Model/Assumption**: In 3 months, the stock price is either $22 or $18 (no dividend for now).

  \[
  S_t = 20 \quad \quad \quad \quad \quad \quad S_T = 22 \quad \quad \quad \quad \quad \quad S_T = 18
  \]

**Comments:**

- Once we make the binomial dynamics assumption, the actual probability of reaching either node (going up or down) no longer matters.

- We have enough information (we have made enough assumption) to price options that expire in 3 months.

- **Remember**: For derivative pricing/hedging, what matters is the list of possible scenarios, but not the actual probability of each scenario happening.
A 3-month call option

Consider a 3-month call option on the stock with a strike of $21.

- **Backward induction**: Given the terminal stock price \( S_T \), we can compute the option payoff at each node, \( (S_T - K)^+ \).

- The zero-coupon bond price with $1 par value is: \( 1 \times e^{-0.12 \times 0.25} = 0.9704 \).

\[
\begin{align*}
B_T &= 1 \\
S_T &= 22 \\
c_T &= 1
\end{align*}
\]

\[
\begin{align*}
B_T &= 1 \\
S_T &= 18 \\
c_T &= 0
\end{align*}
\]

Two angles:

- **Replicating**: See if you can replicate the call’s payoff using stock and bond. ⇒ If the payoffs equal, so does the value.

- **Hedging**: See if you can hedge away the risk of the call using stock. ⇒ If the hedged payoff is riskfree, we can compute the present value using riskfree rate.
Replicating

\[ B_T = 0.9704 \]
\[ S_T = 22 \]
\[ c_T = ? \]

\[ B_T = 1 \]
\[ S_T = 22 \]
\[ c_T = 1 \]

Assume that we can replicate the payoff of 1 call using \( \Delta \) share of stock and \( D \) par value of bond, we have

\[ 1 = \Delta 22 + D1, \quad 0 = \Delta 18 + D1. \]

Solve for \( \Delta \): \( \Delta = (1 - 0)/(22 - 18) = 1/4. \quad \Delta = \text{Change in C}/\text{Change in S}, \text{ a sensitivity measure.} \)

Solve for \( D \): \( D = -\frac{1}{4} 18 = -4.5 \) (borrow).

Value of option = value of replicating portfolio
\[ = \frac{1}{4} 20 - 4.5 \times 0.9704 = 0.633. \]
Assume that we can hedge away the risk in 1 call by shorting $\Delta$ share of stock, such as the hedged portfolio has the same payoff at both nodes:

$$1 - \Delta 22 = 0 - \Delta 18.$$ 

Solve for $\Delta$: $\Delta = (1 - 0)/(22 - 18) = 1/4$ (the same):

$$\Delta = \text{Change in } C / \text{Change in } S,$$ 

a sensitivity measure.

The hedged portfolio has riskfree payoff: $1 - \frac{1}{4} 22 = 0 - \frac{1}{4} 18 = -4.5$.

Its present value is: $-4.5 \times 0.9704 = -4.3668 = 1c_t - \Delta S_t$.

$c_t = -4.3668 + \Delta S_t = -4.3668 + \frac{1}{4} 20 = 0.633$. 

\[ B_t = 0.9704 \]
\[ S_t = 20 \]
\[ c_t = ? \] 

\[ B_T = 1 \]
\[ S_T = 22 \]
\[ c_T = 1 \] 

\[ B_T = 1 \]
\[ S_T = 18 \]
\[ c_T = 0 \]
One principle underlying two angles

- If you can replicate, you can hedge: 
  *Long the option contract, short the replicating portfolio.*

- The replication portfolio is composed of stock and bond.

- Since bond only generates parallel shifts in payoff and does not play any role in offsetting/hedging risks, it is the stock that really plays the hedging role.

- The optimal hedge ratio when hedging options using stocks is defined as the ratio of option payoff change over the stock payoff change — Delta.

- Hedging derivative risks using the underlying security (stock, currency) is called *delta hedging.*
  
  - To hedge a long position in an option, you need to short delta of the underlying.
  - To replicate the payoff of a long position in option, you need to long delta of the underlying.
Delta hedging completely erases risk under the binomial model assumption: The underlying stock can only take on two possible values.

Using two (different) securities can span the two possible realizations.
- We can use stock and bond to replicate the payoff of an option.
- We can also use stock and option to replicate the payoff of a bond.
- One of the 3 securities is redundant and can hence be priced by the other two.

What will happen for the hedging/replicating exercise if the stock can take on 3 possible values three months later, e.g., (22, 20, 18)?
- It is impossible to find a $\Delta$ that perfectly hedge the option position or replicate the option payoff.
- We need another instrument (say another option) to do perfect hedging or replication.
Muti-step binomial trees

- A one-step binomial model looks very Mickey Mouse: In reality, the stock price can take on many possible values than just two.

- Extend the one-step to multiple steps. The number of possible values (nodes) at expiry increases with the number of steps.

- With many nodes, using stock alone can no longer achieve a perfect hedge in one shot.

- But within each step, there are only two possible nodes. Thus, we can achieve perfect hedge within each step.

- We just need to adjust the hedge at each step — the brilliant idea of dynamic hedging.

- By doing as many hedge rebalancing at there are time steps, we can still achieve a perfect hedge globally.

- In principle, one needs as many (different) contracts as possible scenarios to completely hedge away risk, the beauty of dynamic hedging is to take advantage of the time dimension and use efforts (constantly working to rebalance) to compensate for lack of means (instruments).
The Black-Merton-Scholes (BMS) model

- Black and Scholes (1973) and Merton (1973) derive option prices under the following assumption on the stock price dynamics,

\[ dS_t = \mu S_t dt + \sigma S_t dW_t \]

- The binomial model: Discrete states and discrete time (The number of possible stock prices and time steps are both finite).

- The BMS model: Continuous states (stock price can be anything between 0 and \( \infty \)) and continuous time (time goes continuously).

- BMS proposed the model for stock option pricing. Later, the model has been extended/twisted to price currency options (Garman&Kohlhagen) and options on futures (Black).

- I treat all these variations as the same concept and call them indiscriminately the BMS model.
The key idea behind BMS option pricing

- Under the binomial model, if we cut time step $\Delta t$ small enough, the binomial tree converges to the distribution behavior of the geometric Brownian motion.

- Reversely, the Brownian motion dynamics implies that if we slice the time thin enough $(dt)$, it behaves like a binomial tree.
  
  - Within this thin slice of time interval, we can combine the option with the stock to form a riskfree portfolio, like in the binomial case.
  
  - Recall our hedging argument: Choose $\Delta$ such that $f - \Delta S$ is riskfree.
  
  - Since we can hedge perfectly, we do not need to worry about risk premium and expected return. Thus, $\mu$ does not matter for this portfolio and hence does not matter for the option valuation.
  
  - Only $\sigma$ matters, as it controls the width of the binomial branching.
  
  - Generalization: For perfect hedging, we do not need the dynamics to be GBM per se, but just ask that there be only two possible scenarios at each step.
How effective is the BMS delta hedging in reality?

- If you sell an option, the BMS model says that you can completely remove the risk of the option by continuously rebalancing your shareholdings to neutralize the delta of the option.
- I perform the following experiment using historical S&P 500 options data from Jan 1996 - Jan 2015:
  - Sell a one-month at-the-money call option
  - Record the P&L from three strategies:
    1. **Sell and Hold**: Just hold the option short.
    2. **Static Delta Hedge**: Perform a delta hedge initially, but do no re-hedging.
    3. **Daily Delta Hedge**: Restore the delta to zero at the end of each day.
  - I record the P&L each month, then calculate the variance of P&L for each of the 3 strategies over the 16 year period.
- By what fraction do you think the variance of P&L is reduced as we switch from sell-and-hold to daily delta hedge?
  - [A] No reduction,  [B] 0 to 1/3 ,  [C] 1/3 to 2/3 ,  [D] 2/3 to 100%
Unhedged, the standard deviation of P&L can exceed the option price.

Un-rebalanced, the static delta hedge deteriorates over time, but still removes 70% of the variance over one month.

Rebalanced daily, 95% of the variance can be removed on average over our sample period.

The answer is “D.”
Hedging ATM calls at different maturities and time periods

- Dynamic delta hedge works equally well on both short and long-term call options.
- To gauge the stability of this result over time, we calculate the variance reduction for one month P&L for different calendar years.
- The reduction in variance deteriorates somewhat in times of crisis.
Dynamic delta hedge works equally well on both short and long-term puts.

The reduction in put option variance works well in the recent financial crisis, but some deterioration in early 1997.
Improvements are smaller if the options are too far out of money and never get close to the money, but the variance is small even without hedging.
Effectiveness of delta hedging

- Daily delta hedging with the underlying security can drastically reduce (if not completely remove) the risk of the option position, and is cheap and easy to do in most cases.

- Upon entering an options trade, traders may choose to perform the delta hedge right away or in late afternoons before close (when volume is high) based on their assessment of the situation.

- When the underlying is illiquid and delta hedging becomes costly, traders may also try to achieve “beta” neutrality using a highly correlated but more liquid security.

- Many OTC options trades (such as fx options) are delta-neutral by design.

- OTC currency options are quoted/traded in terms of delta-neutral straddles, butterfly spreads, and risk reversals.

- One can still lose lots of money if one takes the model/method literally and acts as if the hedge is 100% effective (as the theory suggests).

- If one levers up 30 times, a 5% risk becomes 150%.
Sources of delta hedging errors

- Discretization: Hedging error is zero only if hedging updating is continuous.
  - Average hedging error is proportional to $\sqrt{\Delta t}$.

- Calculated sensitivities deviate from actual sensitivities.
  - Different dynamics assumptions imply different sensitivity estimates.

- There are other sources of risk such as volatility risk.
  - One can also perform dynamic vega hedging using options to neutralize the volatility exposure.
  - Since options trading are expensive, vega is often managed within a range instead of completely neutralized every day.
  - Traders often also take deliberate/controlled volatility exposures to gain risk premium.

- Unexpected large moves in the underlying or volatility
  - Dynamic hedging is designed to hedge against small or fixed-size moves, and can break down during unexpected large market movements.
Remedies for discretization error: Hedging frequency

- End-of-day delta neutrality is often required at most large institutions.
- There are researches comparing fixed time frequency ($\Delta t$) versus fixed move ($\Delta S$) updating.
- Practically, it is a combination of both.
  - Neutralize the delta at the end of the day to satisfy risk management requirements.
  - Monitor and hedge intraday if there are large market movements.
- Conceptually, there is a contrast between calendar time and business time
  - Weekend and other nontrading hours amount to little business time
  - Earnings day amounts to 8 days of normal business time.
  - Days with central bank announcements, macroeconomic number releases — many of these come at pre-specified time and hence can be monitored with accuracy.
- The time change between calendar and business also have important implications for option prices: Holding calendar time to expiration fixed, options that cover more business time are worth more.
Remedies on hedging ratios

- The default approach is to use the BMS model with the observed option implied volatility as input to compute delta (and possibly vega) sensitivities.

- Different models imply different sensitivities. Unfortunately, different dynamics can generate the same observed option price behavior.
  - Good model design helps — It is more important for the model to make economic sense.
  - Not only cross-sectional fitting, but also time-series analysis can be useful.

- No matter how hard one tries, there is always an element of model uncertainty.

- Practically, one worries less about being exactly right, but more about being robust — the hedge is about right (approximately) no matter what happens, even if the actual realization deviates greatly from assumed dynamics.
  - The more complicated, the more dramatic the breakdown can be.
  - Be as simple as possible, but not simpler.
Remedies for large, unexpected movements

- Fundamentally, dynamic hedges are designed for small, or fixed-size moves, but are not equipped to handle large unexpected movements no matter how frequent one updates or how one computes the hedging ratios.

- Analysis: Simulate various dynamics, and analyze the portfolio value fluctuation at extreme scenarios

- Alternative hedging ideas: Matching cashflows instead of matching risk sensitivities
  - The good: When cashflow is matched, the risk-sensitivities are also automatically matched, no matter what the underlying dynamics are and what risks are there — robust.
    - Closing the option position is a tautological example of cashflow matching. No position means no risk.
  - The bad: Non-degenerating cashflow matching can be difficult or expensive to achieve.
    - If you can find a cheap, cash flow matching hedge, definitely do it.
    - Smart traders/market makers look out for such opportunities, and take these locked positions out of their risk management book.
Under one-factor Markovian dynamics, one can use a continuum of short-term options to statically and perfectly replicate the payoff of a long-term option.

\[
C(S, t; K, T; \Theta) = \int_0^\infty \left[ \frac{\partial^2}{\partial K^2} C(K, u; K, T; \Theta) \right] C(S, t; K, u; \Theta) dK,
\]

Continuous rebalancing is replaced with a continuum of options. Discretization: Use 3-5 options to approximate the replication.

Simulation evidence:
- Works as well as daily delta hedge under BMS
- Works much better when the underlying jumps.

S&P 500 options: Use 3-5 options to hedge can beat daily delta hedge over a month-long horizon.
Root mean squared hedging error comparison:

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Static with three options</th>
<th>Daily delta</th>
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</thead>
<tbody>
<tr>
<td>Instruments</td>
<td>1</td>
<td>1</td>
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<tr>
<td>Target Mat</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>K</td>
<td>RMSE</td>
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<tr>
<td>100%</td>
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<tr>
<td>110%</td>
<td>0.25</td>
<td>0.50</td>
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</table>
Example: Semi-static hedge of barriers

- Option being hedged: a one-month one-touch barrier option with a lower barrier and payment at expiry
- Hedging instruments: vanilla and binary put options, binary call.
- Procedure:
  - At time $t$, sell the barrier, and put on a hedging position with vanilla options with the terminal payoff
    $1(S_T < L) \left(1 + \frac{S_T}{L}\right) = 2(S < L) - (L - S)^+ / L$
  - If the barrier $L$ never hits before expiry, both the barrier and the hedging portfolio generate zero payoff.
  - If the barrier is hit before expiry, then:
    - Sell a European option with the terminal payoff
      $1(S_T < L) \left(\frac{S_T}{L}\right) = (S < L) - (L - S)^+ / L$. 
    - Buy a European option that pays $1(S_T > L)$.
    - The operation is self-financing under the BSM when $r = q$. 

### Hedging barriers: Dynamic and static hedging performance

#### JPYUSD:

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Mean</th>
<th>Std</th>
<th>Max</th>
<th>Min</th>
<th>Skew</th>
<th>Kurt</th>
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<tbody>
<tr>
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<td>55.40</td>
<td>3.71</td>
<td>67.39</td>
<td>37.77</td>
<td>0.55</td>
<td>2.77</td>
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<td>50.01</td>
<td>0.00</td>
<td>-100.00</td>
<td>-0.02</td>
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<td>Delta</td>
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<td>-138.26</td>
<td>0.79</td>
<td>7.86</td>
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<tr>
<td>Vega</td>
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<td>17.50</td>
<td>67.47</td>
<td>-130.67</td>
<td>1.40</td>
<td>10.14</td>
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<td>18.01</td>
<td>67.47</td>
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<tr>
<td>Static</td>
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<td>10.68</td>
<td>48.94</td>
<td>-68.02</td>
<td>6.66</td>
<td>62.00</td>
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Hedging barriers: Dynamic and static hedging performance

GBPUSD:

<table>
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<th>Strategy</th>
<th>Mean</th>
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<th>Max</th>
<th>Min</th>
<th>Skew</th>
<th>Kurt</th>
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<tbody>
<tr>
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<td>54.98</td>
<td>42.86</td>
<td>0.21</td>
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<td>No Hedge</td>
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<td>Delta</td>
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<td>10.80</td>
<td>49.19</td>
<td>-52.62</td>
<td>5.07</td>
<td>39.81</td>
</tr>
</tbody>
</table>

- Dynamic hedging becomes unstable when the underlying price is close to the barrier.
- The proposed static hedge can handle the situation much better.
- Issue: The derivation of the semi-static hedge is very contract specific, and there is no general rule that one can apply to achieve this purpose.
- Dynamic hedges are much more generally applicable and much easier to derive and implement.
Hedging with nearby contracts

Combine the benefits of both approaches


- Matching sensitivity is generally applicable, but is not that robust (can break down in the presence of large moves or when the computed sensitivity deviates from reality).
- Matching payoffs is robust but has to be derived case by case, thus with limited applicability.
- We propose a new approach that matches sensitivities to contract characteristics (strike, expiry), rather than sensitivities to risk sources, as a way to approximately match the payoff.
  - The linearization (characteristics sensitivity) allows the method to be much more generally applicable than directly payoff matching.
  - Approximately matching payoffs make the method much more robust in the presence of jumps and model uncertainty.
- Simulation and historical analysis show its wide applicability and accuracy.
- In practice, one can incorporate the nearby hedging idea to dynamic hedging... (e.g., only neutralize greeks of nearby contracts)
Application: Systematic options market making

- Let $I$ denote the inventory, $o$ denote the size of an incoming order.
- Let $\alpha$ denote the alpha/edge on the order, derived from an option pricing model.
- Let $\nu_o, \nu_c$ denote the variance matrix of the incoming order and its covariance with the inventory.
  - Constructed/design based on measurements of risk sensitivities, contract similarities, variance/concerns on different risk sources.
  - Construction depends on assumed dynamics and hedging strategies.
- A mean-variance setting can be used to automatically generate the quote spread ($QS$) for each order size of the contract: $QS = \gamma (\nu_o o + \nu_c I) - \alpha$
  - Quotes a higher spread if (i) we are more risk averse ($\gamma$), (ii) the risk of the order ($\nu_o$) is large, (iii) it adds to the risk of the inventory ($\nu_c > 0$), or (iv) it goes against our short-term prediction ($\alpha < 0$).
- The role of liquidity and maturity on QS: Given annualized variance and alpha rate estimates, the actual risk of an order depends on how long one expects to hold the position: The holding period is longer for contracts with spare volume (and/or one-sided order flows), but no longer than expiry.
The idea of dynamic hedging is revolutionary and is what greatly expands the trading of derivatives.

Daily delta hedging of option positions is a must, and intra-day delta hedging is needed in the presence of volatile markets.

Differentiation comes from (i) precise edge from option pricing models and (ii) systematic risk management under both normal and extreme conditions.

- Get the dirty details right: dividend projection, financing/borrow projection, physical-to-business calendar mapping, corporate actions.
- Appropriate design and estimation of simple, robust, economically sensible, and well-performing option pricing models.
- Simulate to understand what can break down and by how much during extreme cases.
- Measure, aggregate/net exposures with nearby contracts.
- Look beyond sensitivities and into payoffs, and take strategic positions to achieve approximate payoff matching.
- Lever delta/volatility comovements across names to reduce hedging costs and expand opportunities...