The Role of Positions and Activities In Derivative Pricing
A discussion of Mixon and Onur “Volatility Derivatives in Practice: Activity and Impact”

Liuren Wu
Baruch College

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The paper investigates how positions and activities from different types of market participants impact the pricing of VIX futures.

**Broad picture and motivation:**
- Vol derivatives activities are concentrated on SPX and activities on the listed *VIX futures* dominate at short term.
- Activities on OTC contracts (variance swaps) are heavier on longer term, and concentrated in the hands of a few institutions.

**Empirical analysis:**
- Regress VIX futures-spot spread for each contract on net dealer positions on that contract, or net demands from asset managers & levered funds — risk effects
- Regress spread changes on position changes — price pressure effects
- Combine the two effects.

⇒ Higher demand leads to higher pricing.
The role of positions/activities in derivatives pricing

- Traditional derivative pricing is based on the principle of perfect replication or equivalently, hedging.
  - It has no implications for risk premium & future forecasts.
  - It has no explanations for open positions & trading activities.

- New developments start to give them explicit roles in derivative pricing.
  - Ross recovery of risk premiums & future forecasts based purely on currently observed option prices.
    - Controversies, debates, and a lot of new research ... Are we inferring “what investors are charging for the risk” or “what they should charge for the expected risk” or neither?
  - Garleanu/Pedersen/Poteshman’s “demand-based option pricing” provides explicit roles for net demands...

This paper provides more empirical evidence on how demand affects derivative pricing — an important topic.
The role of hedging in demand-based derivative pricing

The classic hedging argument should remain an important consideration in risk premium and demand analysis on derivatives.

- When one takes on a derivative position, one does not directly forecast its risk and ask for a compensation for the forecasted risk.
  - Finds the best hedge feasible, and sets the pricing as the hedging cost + a premium on the remaining unhedged risk.
  - If the hedge is perfect (e.g., forward on major currency?), the derivative is priced equal to replication, with no extra information revealed from the derivative itself.
  - In most cases, the price contains information on the pricing (and forecast) of some hard-to-hedge risks (e.g., volatility risk, crash risk...)
  - Premiums on easy-to-hedge risks show up in the pricing of the hedging instruments.

- The role of quantities (demands) can be analyzed analogously...
The classic hedging argument remains an important consideration in demand analysis on derivatives.

- Market microstructure on primary securities (say stock): Market makers adjust quotes on a stock according to order flows on that stock as a function of inventory risk management and information expectation.

- When one takes on a derivative position, one does not just sit on the position and collect risk premium.
  - Hedges with the underlying, thus transferring the trading activities (order imbalances) and price pressure to the underlying.
    - Hu (2014): Stock order flow imbalance coming from delta hedging is more informative than order flows from pure stock trading...
  - Hedges with nearby contracts, thus propagating trading activities and price impacts across the whole surface/curve.
    - Options market makers update the whole implied volatility surface in response to order flows from any one particular contract.
  - These propagations (across strikes, maturities, and related contracts) are what makes quantity analysis in derivatives unique and interesting.

*How far, how much, how fast?*
Propagation in SPX volatility derivatives market

VIX futures is not only the most actively traded vol derivatives on the most active index (hence economically significant), but it is also the hub of many connections, and thus the perfect test bed for demand and pricing propagations.

- The term structure: Hedging a futures contracts with a nearby futures contract generates a propagation of impacts across the term structure.

- Structural relations between SPX options, variance swaps, VIX futures, VIX futures options.
  - (Spot/Forward) $\text{VIX}^2 = \text{Portfolio of SPX options}$
  - Forward $\text{VIX}^2 - \text{VIX futures}^2 \sim \text{Portfolio of VIX futures options}$

- Other related products: SPY options, SPX futures option, ...

- Connections versus segmentations
  - The listed market (VIX futures, listed SPX options) at the short end versus the OTC market (variance swaps) at the long end.
  - How do segmentations inhibit, slow down, or distort propagation along the term structure?
**Capacity constraints and the “risk effect”**

- Risk effect on an individual VIX futures contract:
  \[ \nu_i - \nu_0 = \alpha_i + \beta_i \nu_0 + \gamma_i (Dealer_i) + \varepsilon_i \]

  - **Motivation:** Dealer has capacity constraints, thus inducing a positive relation between pricing and net short position — “risk effect.”

- **Caveat:** Capacity constraint is not on one particular naked contract, but on aggregate exposure (unhedged risk) and leverage (total position).
  - “Net” risk exposure is an important calculation for risk management.
  - Reserve is set according to exposure.
  - Size is constrained by capital/leverage limit.

- Consider the “risk effect” on 2-month futures for the following two scenarios:
  - Net short $200mm on 2-month futures, net short $100mm each on 1-month and 3-month futures, too.
  - Net short $200mm on 2-month futures, net long $100mm each on 1-month and 3-month futures.
An integrated analysis of all VIX futures contracts

- Risk effect on an individual VIX futures contract:
  \[ v_i - v_0 = \alpha_i + \beta_i v_0 + \gamma_i(Dealer_i) + \epsilon_i \]

- Risk effect on the VIX futures term structure:
  \[ v_i - v_0 = \alpha(T_i) - \beta(T_i)v_0 + \gamma \sum_{j} h(T_j - T_i)(Dealer_j) + \epsilon_i \]

  - \( \alpha(T_i), \beta(T_j) \) denote pricing propagation function, e.g.,
    \[ \beta(T) = 1 - e^{-\kappa T}, \alpha(T) = \theta \beta(T). \]
    - \( \kappa \) captures how fast price shocks propagates along the term structure.
  - \( h(T_j - T_i) \) is a demand propagation function, e.g.,
    \[ h(T_j - T_i) = e^{-\eta |T_j - T_i|}. \]
    - \( \eta \) captures how fast quantity shocks propagates along the term structure.

- How to best aggregate net exposure is up to future research...

- We know the direction of the impact (the higher, the higher)... More interesting is the propagation function: How far, how much, how fast?

- The identification should also be stronger on aggregate net exposure.
Link to listed options and OTC variance swaps

- Risk effect on one VIX futures: \( v_i - v_0 = \alpha_i + \beta_i v_0 + \gamma_i (Dealer_i) + \varepsilon_i \)

- Risk effect on the VIX futures term structure:
  \( v_i - v_0 = \alpha(T_i) - \beta(T_i) v_0 + \gamma \sum_j h(T_j - T_i)(Dealer_j) + \varepsilon_i \)

- Risk effect on the SPX options term structure (in parallel):
  - Synthesize forward VIX \(^2\) from SPX options at longer maturities.
  - Aggregate demands on listed SPX options
  - Compare VIX futures term structure to forward VIX term structure.
    - There is a gap in level due to variance of volatility, but should there a same/similar term structure effect?
    - VIX futures options can be used to link the two.
  - Aggregating net exposure across related contracts should be even more interesting...

- Extend the variance swap term structure to OTC range (longer maturities)
  - How strong is the segmentation between listed market and OTC?
  - How does the concentrated (oligopoly?) nature of the OTC positions affect the propagation?
Link price pressure (transition) to risk effect (equilibrium)

- Risk effect: \( v_i - v_0 = \alpha_i + \beta_i v_0 + \gamma_i(Dealer_i) + \varepsilon_i \)

- Price pressure on the risk effect:
  \[ \Delta(v_i - v_0) = \alpha_i + \beta_i \Delta v_0 + \gamma_i(\Delta Dealer_i) + \varepsilon_i \]

- These two effects can be integrated
  - Open interest is a unique feature of derivatives and reflects the demand (non-redundancy) of the market.
  - If there indeed exists a level relation between pricing and demand, demand change reflects transitions from one level relation to another.
  - If adjustment takes time, one can use an error-correction specification to forecast the future changes in price and quantity:
    \[ \Delta(v_i - v_0) = \zeta_1(v_i - (1 + \beta_i)v_0 - \alpha_i - \gamma_i(Dealer_i)) + e \]
    \[ \Delta(Dealer) = \zeta_2(v_i - (1 + \beta_i)v_0 - \alpha_i - \gamma_i(Dealer_i)) + e \]

- Such forecasting analysis becomes more interesting, and probably more accurate, when the co-integrating relation is built on aggregate net risk exposure.
Concluding remarks

- Traditional derivative pricing is based on perfect replication/hedging and treats derivatives as redundant securities, thus generating no implication for risk premium analysis and no explanation on their demands.

- This paper documents the demands and activities on volatility derivatives and shows empirical linkages between demand and pricing — an important consideration for future derivative pricing development.

- These new considerations will not replace classic hedging arguments, but will build upon them.
  — Hedging plays an important role in analyzing demands and risk premiums with derivatives.

- Demands for derivatives and their hedging instruments need to be analyzed together to understand how individual demand propagates through hedging activities, and how aggregate net demand affects equilibrium pricing.

- Look forward to more future research along these promising and interesting directions.